UNIVERSITY OF CALGARY

Essays on the Economics of Geoengineering

by

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A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF ECONOMICS

CALGARY, ALBERTA
JANUARY, 2010

THE UNIVERSITY OF CALGARY
DEPARTMENT OF
ECONOMICS

The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "**Essays on the Economics of Geoengineering**" by **Juan B. Moreno-Cruz** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy**.

Dated: December 2009

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To Lindsay, the love of my life;

and to the joyful gift she is about to bring home.

A mi Pa y a mi Ma por su apoyo y amor incondicional.

A la Nena, Pablo y Diego, por su complicidad.

To God, for the endless opportunities.

Table of Contents

Abstract

It appears technically feasible to engineer an increase in albedo, a planetary brightening, as a means to offset the warming caused by CO_2 and other greenhouse gases through Solar Radiation Management (SRM). This possibility has created an intense debate given the ethical, moral and scientific questions it raises.

The aim of this work is to examine the economic issues introduced when geoengineering becomes available to deal with climate change. I study the implications of geoengineering on the design of optimal climate change policies (certainty and uncertainty scenarios), and also the strategic interactions created when geoengineering becomes available in an model with active, but suboptimal abatement.

Acknowledgements

I would like to thank David W. Keith, my supervisor, for his many suggestions and constant support during this research; his curiosity is a constant source of inspiration. I also want to thank Sjak Smulders, my co-supervisor, for his guidance through the early years of confusion; for helping me set my priorities right. Scott Taylor help me during the development of my third paper. His guidance improved, not only my work, but my understanding of economics as a whole.

I want to specially thank Dr. Scott Barrett for agreeing to be the examiner of my thesis. I hope he finds it of interest.

I want to thank Lindsay, my wife, for her understanding and patient love. She is the strength in my life and the reason this thesis is finished. I also want to thank my baby, because her arrival defined the deadline for my defense and push me to finish sooner.

Of course, I am grateful to my parents for their patience and love. Without them this work would never have come into existence (literally). I thank my sister, Nena, and my brothers, Pablo and Diego, for their unlimited support and their strong believe on my success. I also want to thank my family here, for welcoming me in your lives.

Finally, I wish to thank the following: Dan (for his friendship); Jesse (for so many coffees); Joule and Jeff (for their friendship and for Liam); Jevan (for keeping up to date with the independent music scene); and to my good friend Hisham, who saw the potential (good luck with parenthood!).

Calgary, Alberta — Juan B. Moreno-Cruz
January 14, 2010

Acronyms

BAU	Business As Usual
CDR	Carbon Dioxide Removal
GDP	Gross Domestic Product
GHG	Greenhouse Gases
SRM	Solar Radiation Management

Introduction

The Doomsday machine is terrifying and simple to understand...and completely credible and convincing.

Dr.Strangelove

Considering the possibility of a climate crisis and the inability of nations to coordinate effectively on the climate change problem, scientists are now exploring a new set of technologies designed to quickly lower temperatures without lowering greenhouse gas concentrations. These technologies fall under the category of geoengineering. The literature distinguish two modes of geoengineering: Solar Radiation Management (SRM) and Carbon Dioxide Removal (CDR). These modes differ in two principal ways. First, a reduction in the incoming radiation has almost instantaneous effects on global temperature—after Mount Pinatubo's explosion in 1991 global surface temperatures cooled about 0.5^{o}C over the following year. CDR is necessarily slower, because the removal flux grows at the rate at which the capital stock of CDR is accumulated, and the reduction in CO_2 concentration grows as the integral of the flux. Thus, there is a long time delay between commitment to implement CDR and any significant reduction in climate impacts. Second, SRM cannot exactly reverse the climate change caused by greenhouse gases. A reduction in insolation using SRM produces climate change that partially counter the climate change due to greenhouse gases. For example, if SRM is used to bring global average temperatures back to the preindustrial level it will tend to over control precipitation reducing it below preindustrial means and will do little to alter the impact of CO_2 on ocean acidification. Conversely, CDR can, ignoring hysteresis, fully correct CO_2 driven climate change, although any given CDR method may cause other environmental problems. Here, I concentrate on SRM because this response mode differs most fundamentally from emission mitigation; in what follows I use the terms geoengineering and SRM interchangeably.

Throughout my thesis, I try to capture the main characteristics of geoengineering that matter most to inform policy and governance decisions; the same characteristics that make geoengineering both, incredible and dangerous. In particular, my analysis aims at a parsimonious representation of the following stylized facts: First, *SRM is imperfect*; in order to fully eliminate the effects of climate change we need a combination of abatement and SRM. Second, *SRM is fast*; which implies that we can wait to observe the magnitude of the damage from climate change and then implement some level of SRM appropriately. Third, *SRM is cheap*; which makes SRM essential to achieve any temperature target at lower costs relative to traditional mitigation. Finally, the effectiveness and side effects of *SRM are uncertain.* The intentional modification of the earth's climate may lead to nonlinearities in the system creating even greater problems: changes in precipitation, ozone depletion, and health issues.

My thesis is a compilation of three self-contained papers. Each paper answers important, but distinct, questions about the interaction of abatement and geoengineering and the relevance of these technologies in managing the climate; taking into consideration the trade-off between the advantages and disadvantages of implementing geoengineering.

In the first chapter, "Revisiting the Economics of Climate Change: The Role of Geoengineering," written in close collaboration with Dr. Sjak Smulders, geoengineering is introduced into a canonical economic model of climate change. I show that because geoengineering technologies deal with the impacts of climate change, but not with the source of the problem (increasing carbon emissions), they cannot fully replace strategies leading to emissions reductions. However, geoengineering can be used as a complement to traditional abatement strategies to stabilize atmospheric carbon concentrations at a much lower cost to society relative to a policy without geoengineering. I also find that for low geoengineering costs, a negative carbon tax cannot be ruled out theoretically. However, a simple calibration of the model shows that the possibilities of a negative carbon tax are very slim.

Once the main framework is understood, I modify it to analyze questions of uncertainty and risk, as well as issues of strategic interaction. Specifically, in the second chapter, "Climate Change Policy under Uncertainty: A Case for Geoengineering," written in close collaboration with Dr. David Keith, the framework developed in the first chapter is expanded to allow for uncertainty on the effectiveness and the consequences of geoengineering. In this chapter I find that imperfect geoengineering is an effective means to approach the uncertainty in the climate response because it can be implemented after this uncertainty is resolved, providing a tool to manage the inertia in the carbon-climate decision problem. Without geoengineering, the high-consequence low-probability climate events drive very high levels of abatement due to the irreversibility of their impacts. In this simplified model, geoengineering is used in the case of an unlucky (high-impact) outcome even if the damages from geoengineering exceed the damages from global warming, and even if geoengineering is not very effective. Under similar assumptions, the use of geoengineering is substantially reduced when climate impacts are relatively low.

When uncertainty on the geoengineering option is introduced, I find that learning about geoengineering — that is the value of information associated with reducing the uncertainty about the effectiveness and side effects of geoengineering — may be worth several trillion dollars over the next 100 years. This specific numerical result depends, of course, on the calibration of the model and on the assumptions about the prior probability distribution over the effectiveness of geoengineering in managing climate impacts. I used a calibration of the economics of climate damages and abatement that is representative of results derived in many complex models. In addition, I assumed the expected damages from geoengineering greatly exceed current estimates; I therefore expect the general result that the value of significantly reducing the uncertainty about geoengineering exceeds several trillion dollars is robust. A secondary contribution of our paper is that geoengineering and abatement are *risk complements*: The riskier is the prospect of geoengineering the higher is the level of abatement implemented in the optimal policy; thus, abatement becomes an insurance against risky geoengineering.

This treatment of uncertainty is limited as it does not capture the possibility of catastrophic effects due to greenhouse gases accumulation or geoengineering implementation. Catastrophic outcomes are those defined as singletons in time that are unique both in the way they occur and the large implications they have on the economy and the ecosystem. These issues cannot be addressed with the parsimonious framework developed here. Further work in this areas should be pursued.

In the third chapter, "The Long and Short of Climate Change: Abatement versus Geoengineering," I expand the framework developed in the first chapter in a different direction. This chapter examines the strategic issues introduced when geoengineering becomes available as a new instrument to combat climate change in an otherwise standard two country model with active, but suboptimal, abatement. Geoengineering naturally introduces the possibility of technical substitution away from abatement. This technical substitution reduces the overall costs of managing climate change and is the correct response given this specific framework. In particular, independent of the strategic environment, it is in each country's best interest to reduce its level of abatement and increase its level of geoengineering given the assumption of increasing and convex costs. Geoengineering also affects the strategic interaction between countries. Specifically, considering the global public good nature of abatement and geoengineering, and the substitutability of both technologies in reducing global temperature, it is possible that countries will substitute away from abatement to induce higher levels of geoengineering in other countries.

However, contrary to intuition, once the full range of strategic effects is considered, geoengineering may induce a level of abatement that is higher than the first best level of abatement in the absence of geoengineering. In particular, if the damages caused by the unintended consequences of geoengineering differ across countries, abatement in some countries would increase in order to deter the use of geoengineering by other countries. It seems that other deterrence instruments, not captured in the model, could be more effective at inhibiting the use of geoengineering: trade sanctions and direct conflict seem to be more plausible. The presence of this more direct actions to limit the use of geoengineering make the counterintuitive result described above less likely.

Another interesting possibility, yet to be explored, is that of a "stratospheric war" in which countries engage in a series of geoengineering and counter-geoengineering experiments jeopardizing the stability of the climate and the political systems. This type of interaction cannot be studied using the current framework. The analysis of the implications that this possibility has on the governance of geoengineering is left for future research.

Chapter 1

Revisiting the Economics of Climate Change: The Role of Geoengineering

Technically simple and reversible measures to directly reduce mean global temperatures could be available anytime soon. These geoengineering technologies could combat global warming at low cost. Injecting small particles in the stratosphere is the most promising example, but there are other ones. We introduce the concept of "geoengineering" into an analytical model of climate change. We model the technical and economic characteristics of geoengineering in line with the recent geoengineering literature from physical and environmental management sciences. We investigate (i) under which circumstances geoengineering can substitute, partly or completely, for traditional abatement strategies, (ii) under which conditions and at what level geoengineering is optimally employed, and (iii) whether geoengineering can mitigate free-riding problems. [1]

[1]This chapter is jointly written with Sjak Smulders. We thank Paco André, Rob Hart, David Keith, Timo Goeschl, Mark Jaccard, Robert Oxoby, Daan van Soest, Scott Taylor and participants at the Grenoble Workshop on Innovation and the Environment, the IEW 2007 meeting at Stanford, and the UNU-MERIT research seminar for comments on a previous version.

1.1 Introduction

Scientists have recently worked on, discussed, and experimented with several technologies that can relatively easily alter global environmental systems. Elements of these technologies, which come under the label of "geoengineering", have been put in practice already, e.g. as weather modification at a local scale to boost agricultural productivity or ensure a clear sky at celebrations like the 60th anniversary of the People's Republic of China. Geoengineering technologies sometimes appear to be taken from a Jules Verne book: they range from giant mirrors in the space to block the sun (Angel, 2006), to artificial volcanoes that mimic the idea of a giant volcanic eruption releasing huge amounts of sulfur into the atmosphere - as Mount Pinatubo did in 1991 with a point-five-degree-Celsius cooling effect (Soden et.al. 2002). However, far from fiction, the scientific principles behind geoengineering technologies are well established (Caldeira and Wood, 2008). Moreover, it has been suggested that the cost of geoengineering could be so low compared to traditional mitigation strategies that they would make climate change irrelevant.

In this paper we analyze the economics of geoengineering and its implications for climate change policies that follow from what we consider to be the main characteristics of a geoengineering technology. In particular, we investigate (i) under which circumstances geoengineering can substitute, partly or completely, for traditional abatement and mitigation strategies, (ii) under which conditions and at what level geoengineering is optimally employed, and (iii) whether geoengineering can mitigate free-riding problems. To answer this questions we include the idea of geoengineering into a prototype economic model of climate change. The main difference with the standard approach is that we have to disentangle the damages from temperature changes from those of the concentration of greenhouse gases (GHG).

Free-riding, governance, and uncertainty aspects are at the heart of the geoengineering problem. However, we should first clearly set out the concept of geoengineering, define its characteristics, and explore its fundamental impacts on the economy before we can deal with the more complex issues. This is exactly the aim of our paper: to develop the framework in which geoengineering can be studied.

Given the costs of geoengineering appear to be low relative to mitigation strategies, it could be possible for a single country to intervene and unilaterally implement geoengineering. At the end of the paper, we extend our model to study the effects that the unilateral implementation of geoengineering would have in a multi-country setting in which abatement is implemented at sub-optimal levels.

The series of commentaries on Nobel Prize winner Paul Crutzen's article in Climatic Change (2006) revealed the multiple questions surrounding geoengineering. It comes across that whenever promoted, research on geoengineering is only proposed as an insurance against an abrupt climate change happening in the future (see for example Schneider, 1996 and Cicerone, 2006). However, before we can discuss the risk over possible scenarios, we need to know what are the qualitative effects of geoengineering that will be common in these scenarios. It is this question on the nature of the economic implications of geoengineering that we want to address in this paper, which is of independent interest of (and should be addressed before) the question of insurance.

We define geoengineering as an action in which the primary objective is the modification of the climate system at a large scale (Keith, 2000). Although this definition is broad and apply to several technologies, we have a particular one in mind for the sake of concreteness, viz. solar radiation management, aiming to counter effect the increases in temperature due to the increase in GHG concentrations. Given the varying tradeoffs between up-front costs, effectiveness, and adverse environmental consequences, this technology appears to be optimal in a scientific sense (MacCracken, 2006, and Lenton and Vaughan, 2009).

The uncertainties surrounding the implementation of geoengineering, as well as the ethical questions around large scale manipulation of the environment, have kept geoengineering technologies under the radar of the climate change community. Nonetheless lately, scientists have argued that geoengineering needs to be considered as a potentially serious player in the climate change arena, and research on the use and impacts of these technologies must be promoted (American Meteorological Association, 2009).

From a technology perspective, geoengineering is distinct from traditional climate change mitigation strategies (abatement, for short) due to three particular reasons: first, the effects of geoengineering are immediate (Mathews and Caldeira 2008) while abatement is slow because it relies on the carbon cycle. Second, geoengineering sometimes has the connotation of fixing the climate change problem at zero (or very low) cost and thus making climate change irrelevant (cf the scenario that Nordhaus and Boyer, 2000, label geoengineering). However, in our model we acknowledge the fact that geoengineering is not a perfect substitute for abatement measures: rather than reducing greenhouse gases *emissions*, geoengineering reduces the *impacts* of the concentration level of these gases in the atmosphere. Geoengineering cools down the planet and reduces the temperature-related damages, while other problems derived from GHG concentration are not corrected (e.g. ocean acidification). It can also introduce new damages on its own (e.g. precipitation changes (Bala et.al. 2008), or health related issues (Crutzen 2006)). Third, geoengineering is much less costly than abatement and a single country could easily undertake enough geoengineering to protect the entire planet against the damage from global warming (Keith and Dowlatabadi 1992, Crutzen 2006, Blackstock et.al. 2009, Shepherd et.al. 2009). In contrast, traditional abatement strategies rely on the inertia of the carbon cycle and they effects are are prone to international coordination and free-riding problems (Schelling, 1996).

In the economics literature, Wigley (2006) provides the first discussion of the implications of combining traditional mitigation and geoengineering. He compares different levels of geoengineering in different emission scenarios, but does not allow for interaction between geoengineering levels and emission levels: both are exogenously and independently fixed. Schelling (1996) provides the first discussion of strategic effects of geoengineering. He warns against the possibility of conflict arising from the fact that geoengineering is cheap enough to be implemented by a single country (or a small coalition of countries), while the impacts of geoengineering are global and unevenly distributed among countries. Barrett (2008) highlights the characteristics that make geoengineering unique, and warns against the negative effects of its implementation due to the increase in free-riding on abatement. He argues that it is essential to study free-riding issues and governance systems that deal with the question of who should be allowed to start using geoengineering, when and under which conditions. Along the same lines, Victor (2008) argues that new regulation is needed to control the use of these technologies. He also argues that although the implementation cost may be low, new costlier interventions will be needed to off-set possible negative effects, and these costs should be considered when designing policies that include geoengineering options.

The rest of the paper is organized as follows. The next section explains which of the canonical elements of traditional climate change models needs to be changed in order to incorporate geoengineering. Then, we analyze optimal choices of abatement and geoengineering. Finally we turn to a second-best multi-country setting to study free-riding behavior.

1.2 Geoengineering and the Climate-Economy Link

Geoengineering is aimed at reducing the intensity with which the sun warms the planet. Traditional models of climate change assume that there is a direct mapping from concentration of CO_2 to mean global temperature. However, once we consider geoengineering we have to disentangle the damages from temperature (e.g. sea level raise) from those of CO_2 concentration (e.g. ocean acidification), because geoengineering affects the relation between CO_2 concentrations and temperature. Also, when considering the amount of CO_2 emissions that end up in the atmosphere, we must separate the effects of temperature and CO_2 concentrations on the global carbon cycle. In this section we first define temperature as a function of atmospheric concentrations of CO_2 and geoengineering, second we study the carbon cycle and, third we study the economic damages.

1.2.1 Radiative Forcing and Temperature

Radiative forcing is defined as the change in the balance between incoming shortwave radiation and outgoing longwave radiation caused by human activity (IPCC, 2007). The relation between temperature changes, T, and radiative forcing, R, is linear, and can be written as:

$$T = \lambda R$$

where λ is a constant known as climate sensitivity. Radiative forcing increases with the concentration of CO_2 in the atmosphere, S, and decreases with geoengineering, G. We assume that the relation between CO_2 and radiative forcing is given by $P(S)$, with $P'(S) > 0$ (see Lenton and Vaughan, 2009). The function $P(S)$ is defined by the IPCC 2007 as a logarithmic function of the form:

$$P(S) = \beta \ln \left(\frac{S}{S_0} \right)$$

where S is measured in parts per million (ppm), S_0 is the preindustrial reference concentration and $\beta = 5.35Wm^{-2}$. If we define geoengineering, G, in terms of its radiative forcing potential, then temperature can be written as:

$$T = \lambda \left(\beta \ln(S/S_0) - G \right) \equiv t(S, G) \tag{1.1}$$

CO_2 concentrations increase temperature, which we interpret as the global warming effect, $t_S(S, G) > 0$, while geoengineering produces a cooling effect, $t_G(S, G) < 0$. Taking $P^{''}(S) = -\beta/S^2$ and for given $S > S_0(= 280ppm)$, we have $P^{''} \sim 10^{-5}$. Thus, for concentration levels higher than preindustrial levels, we can assume that $t_{SS}(S, G) \approx 0$. Given the linear relation between radiative forcing and geoengineering, and the separability of S and G in t, we have that $t_{GG}(S, G) = 0$, and $t_{SG}(S, G) = 0$.

1.2.2 The Carbon Cycle

The amount of carbon in the atmosphere is determined by carbon emissions and carbon uptake from the atmosphere by the biosphere. The total net carbon uptake (or absorption capacity) of the biosphere, denoted by δ, is itself a function of CO_2 concentrations, S, temperature, $T = t(S, G)$, and geoengineering, G; and it is given by:

$$\delta(S, G) = \delta^1(S, t(S, G)) + \delta^2(G) \tag{1.2}$$

$$\delta^1_S(S, T) > 0,\ \ \delta^1_{SS}(S, T) < 0$$

$$\delta^1_T(S, T) < 0,\ \ \delta^1_{TT}(S, T) < 0$$

$$\delta^1_{ST}(S, T) < 0.$$

$$\delta^2_G(G) \approx 0,\ \ \delta^2_{GG}(G) \approx 0$$

The Earth's biosphere and atmosphere balances the incoming shortwave solar radiation with longwave terrestrial radiation leaving the Earth. This process is the combination of, on the one hand, carbon sequestration by plants and oceans, and, on the other hand, heterotrophic respiration. Oceans and plants take up more CO_2 the more of it there is in the atmosphere. This causes the net primary productivity to increase. Thus, for given temperature, carbon uptake increases with CO_2 concentration. We capture this by assuming $\delta^1_S > 0$.

More carbon is released from plants at higher temperatures due to an increase in heterotrophic respiration (Matthews et al. 2005). Hence, for any given CO_2 stock, carbon uptake increases with temperature, which we capture by the term $\delta^1_T < 0$. Moreover, computer experiments using *general circulation models* have shown that the effects of temperature in the absorption capacity of the atmosphere decrease at higher concentration levels of CO_2. That is, $\delta^1_{ST} < 0$ (Canadell et.al. 2007).

Geoengineering decreases the amount of shortwave radiation which decreases carbon uptake. However, it also increases the diffusion of the radiation which in turn increases the productivity of plants at capturing CO_2. The net effects on the absorption capacity, captured by $\delta^2(G)$, are expected to be small relative to the direct temperature effects (Govindasamy et al., 2002). This allows us to impose the condition $\delta^2(G) \approx 0$ and neglect the last term at the right-hand side of (1.2). Thus, geoengineering affects absorption capacity only through its effects on temperature.

We model the reduction in atmospheric CO_2 relative to "Business As Usual" level as a result of an increase in carbon uptake, δ, or a reduction in emissions, E. In particular, atmospheric CO_2 reduction $(S^{BAU} - S)$ equals the sum of additional uptake $(\delta - \delta^{BAU})$ and emissions reduction $(E^{BAU} - E)$, where the BAU superscript denotes the exogenous "Business As Usual" levels, i.e. levels that would arise in the absence of climate change policy:

$$(S^{BAU} - S) = (\delta - \delta^{BAU}) + (E^{BAU} - E)$$

Substituting (1.2), collecting the relevant BAU terms in $\bar{S}^{BAU} \equiv S^{BAU} + \delta(S^{BAU}, 0)$, and defining abatement as emissions reduction $A \equiv E^{BAU} - E$, we may write the above equation as:

$$\bar{S}^{BAU} = S + \delta(S, G) + A \tag{1.3}$$

Equation (1.3) defines the level of abatement, A, needed to reach a certain level of atmospheric concentrations of CO_2, S, given the amount of geoengineering, G.

Our model is static in the sense that we are concerned with atmospheric concentration levels at one particular (future) date only, as the result of (cumulative) emissions and (cumulative) absorption up to that date (these three variables are denoted by S, E and δ respectively); we are not concerned with either the timing of emissions or the dynamics of the carbon cycle. Dynamic modeling is left for future research.

The implications of our modeling of temperature and carbon cycle, equations (1.1) and (1.2), are summarized as follows

Lemma 1: *For all S and G, $\delta_S > 0$, $\delta_{SS} < 0$, $\delta_G > 0$, $\delta_{GG} < 0$, and $\delta_{SG} > 0$.*

Proof: follows from (1.1) and (1.2). See appendix for details.

Hence, the carbon sink increases (at a decreasing rate) with CO_2 and with geoengineering. The latter is solely due to the effect of geoengineering on temperature: geoengineering results in lower temperature so that the terrestrial biosphere absorbs more carbon.

1.2.3 Damages and Geoengineering Intervention

Damages caused by increases in temperature come in the form of increased precipitation, more unpredictable weather patterns and storms, sea-level rise, desertification and loss of fertile soils, etcetera. The key feature of geoengineering that makes it so promising to combat climate change problems is that by increasing the reflectivity of the atmosphere (albedo), more incoming sunlight is reflected back and temperature on Earth is reduced. Hence, geoengineering can undo damages from climate change. However, this is not to say that geoengineering neutralizes all effects of CO_2 concentrations. Although it can keep the temperature below dangerous levels and thus address global warming damages, it cannot, for example, avoid the effects on the acidification of the oceans caused by CO_2 accumulation.

To clarify our argument, we specify these damages in a way that separates temperature effects from other effects. In particular, as we did with the absorption capacity function, we assume that damages, D, depend on atmospheric concentrations of CO_2, S, and geoengineering, G, in the following way:

$$D(S,G) = d^1(S) + d^2(t(S,G)) + d^3(G); \tag{1.4}$$

$$d^1_S(S) < (>)0 \text{ for } S < (>)\bar{S};\ d^1_{SS}(S) > 0;$$

$$d^2_T(T) < (>)0 \text{ for } T < (>)\bar{T};\ d^2_{TT}(T) > 0;$$

$$d^3(0) = d^3_G(0) = 0;\ d^3_{GG}(G) > 0.$$

The first term at the right-hand side of (1.4) represents the idea that CO_2 concentrations have effects other than temperature changes, which are on balance positive for small concentration levels and negative for large ones (Matthews et. al., 2005). The positive effects stem from a fertilization effect: more CO_2 enhances plant growth and thus boosts productivity in agriculture (King et. al., 1997). The negative effects stem from acidification of oceans and other ecological disruptions which ultimately harm productivity of the economy (Caldeira and Wickett, 2005). Since the marginal fertilization effect is decreasing (Cao and Woodward, 1998 and King et.al. 1997), and the marginal acidification effect is likely to be increasing in S, we assume a critical level $\bar{S}$ exists such that, for given temperature, CO_2 accumulation turns from a benefit into a damage.

The second term at the right-hand side of (1.4) captures the effect of global warming. Excessive increases in temperature (as well as excessive cooling, i.e. temperature below $\bar{T}$) increase damages. We assume that marginal damages from temperature, whether positive or negative, always rise with temperature ($d^2_{TT}(T) > 0$). At some places in the paper it is relevant to know if these marginal temperature effects strongly increase. For this reason we formulate the following assumption, which we will explicitly refer to if we use of it:

Assumption DT (strong temperature effects): $d^2_{TT}(T) > \beta\lambda d^2_T(T)$ for all T.

The third term at the right-hand side of (1.4) captures the damages caused by geoengineering alone. Recent numerical simulations on the use of geoengineering show that geoengineering will affect precipitation patterns and volumes, causing a decrease in precipitations over land and an increase in precipitations over the ocean (Matthews and Caldeira, 2007), possibly causing droughts in large regions of the planet. Our assumption is that more geoengineering entails more damage, whether the aim is cooling ($G > 0$) or warming ($G < 0$).

Because of the second term at the right-hand side of (1.4), it is relevant for marginal damages whether temperature is below or above $\bar{T}$. Therefore we define the critical level $S^{\bar{T}}$ such that in the absence of geoengineering and with S at this level, the marginal damages from temperature change are exactly zero (i.e. if $G = 0$ and $S = S^{\bar{T}}$ then $T = \bar{T}$). From (1.1) we find:

$$S^{\bar{T}} \equiv S_0 \exp(\bar{T}/\lambda\beta)$$

The implications of our modeling of temperature and damages, equations (1.1) and (1.4), are summarized as follows

Lemma 2:

(a) under assumption DT, a unique value $S^{ZDN} \in [\min\{\bar{S}, S^{\bar{T}}\}, \max\{\bar{S}, S^{\bar{T}}\}]$ *exists such that* $D_S(S, 0) > 0$ *if and only if* $S > S^{ZDN}$*;*

(b) a function $\Gamma(S)$*, with* $\Gamma'(S) > 0$ *and* $\Gamma\left(S^{\bar{T}}\right) = 0$*, exists such that* $D_G(S, G) < 0$ *if and only if* $G < \Gamma(S)$*;*

(c) for all S *and* G*,* $D_{GG} > 0$ *and* $D_{SG} < 0$*;*

(d) under assumption DT, $D_{SS} > 0$ *for all* S *and* G*.*

Proof: follows from equations (1.1) and (1.4); see appendix for details.

In words, the lemma states the following. Given geoengineering levels, CO_2 accumulation reduces damages at first (i.e. as long as $S < S^{ZDN}$), but is damaging (at an increasing rate) for higher levels (i.e. when $S > S^{ZDN}$). This reflects the acidification effect. Additional geoengineering reduces damages up to a certain point (i.e. as long as $G < \Gamma(S)$), and provided temperature is not at $\bar{T}$. Too high levels of geoengineering result in direct damages from geoengineering that dominate the effect on temperature reduction.

In the absence of geoengineering and for CO_2 levels close to their historical average over the past few centuries, the benefits from fertilization are assumed to be lower than the damages caused by the global warming effect, $d_T^2 t_S > -d_S^1$. Under these circumstances CO_2 accumulation is harmful in the absence of geoengineering. This is the natural assumption to make when studying climate change: without this assumption there was no climate change problem. At currently observed mean global temperature, as well as at the temperature at the start of the industrial age, damages increase by global warming, rather than by global cooling. However, for (substantially) lower CO_2 concentration levels and correspondingly lower temperature, damages from global warming might be no longer relevant and might be replaced by damages from global cooling. Hence, when S decreases sufficiently far from current levels, we would enter the range in which the inequality is reversed, $d_T^2 t_S < -d_S^1$, and CO_2 accumulation would be beneficial rather than harmful. In Lemma 2, this reversal of the effect of accumulating CO_2 in the absence of geoengineering is formalized through the critical level S^{ZDN}: above this level CO_2 accumulation and global warming are damaging on balance, while below this level global warming and CO_2-fertilization are good on balance.

With respect to the net effects of geoengineering, it is expected that the benefits from the cooling effect of geoengineering outweigh the costs from changes in droughts and precipitation. Although this outcome is extremely uncertain, since it is based on computer simulations rather observed in real data, we reflect this outcome in our model by assuming that, for sufficiently high temperature (at least bigger than $\bar{T}$), the marginal benefits of geoengineering exceed the marginal damages ($d_G^3 < -t_G d_T^2$). This is the natural assumption to make in a study on geoengineering when decisions are centralized.[2] With the opposite assumption, geoengineering is never beneficial and can always be ignored as an option in climate change policy. Lemma 2 formalizes this: sufficiently small geoengineering levels (i.e. levels below Γ) are effective in reducing damages.

Temperature rises with S, but at a diminishing rate, due to the logarithmic specification in (1.1). Hence, marginal damages only increase with S over the whole range of S if damages are sufficiently convex in temperature T. Assumption DT is a sufficient (but unnecessarily strict) condition to ensure this result.

[2]In a decentralized equilibrium it is possible to have positive levels of geoengineering even if the condition ($d_G^3 < -t_G d_T^2$) is violated. Moreno-Cruz (2009) analyses a decentralized version of the model.

1.2.4 Costs of abatement and geoengineering

To avoid climate and CO_2 related damage, two strategies stand out. First, atmospheric CO_2 concentration, S, can be reduced (relative to its Business As Usual level, S^{BAU}) by reducing emissions (by amount A). Second, geoengineering, G, can be undertaken to reduce temperature-related damages. The previous section has pointed out the differences in impact between abatement and geoengineering on damages: different from abatement, geoengineering intervention does not affect the level of CO_2 concentrations directly; it decreases damages in the economy by reducing the radiative forcing for any given level of CO_2 concentrations.

We specify the economic, non-climate related, costs K of the two climate change policy tools. In particular, we specify the costs $C(A)$ and $M(G)$ as the cost of implementing and operating abatement and geoengineering, respectively, that society incurs on top of the climate related damages D.

$$K(A, G) = C(A) + M(G); \tag{1.5}$$

$$C_A(0) = 0;\, C_{AA}(A) > 0;$$

$$M_G(0) = 0;\, M_{GG}(G) > 0.$$

Our assumptions on the cost of abatement will be standard: we assume that the marginal cost of abatement is a strictly increasing, strictly convex function of abatement, $C(A)$, where $C_A > 0$ if and only if $A > 0$, and $C_{AA} > 0$ for all A.

With respect to the costs of geoengineering, the technical operation costs are small and largely independent of the amount of cooling to be produced (D. W. Keith, 2000). Nevertheless, there may be non-zero increasing marginal costs associated with geoengineering, mainly outside the realm of operating costs. These costs arise from the effects that geoengineering can have other than those on temperature. We think these are mainly health effects, following Crutzen (2006), who warns for the health and air quality effects of SO_2 concentrations at low levels, and assuming that this can be extended to any type of particles used to increase albedo. We denote the costs of geoengineering by $M(G)$, with $M(0) = 0$, and assume that the marginal costs of geoengineering are increasing and convex, $M_{GG} > 0$ for all G.

1.3 Abatement and Geoengineering Policies

We now combine the absorption function, given by (1.2), and the damage function, given by (1.4), with assumptions on the costs of abatement and geoengineering, given by (1.5), to build a model of climate change economics. To provide the simplest possible benchmark model, we ignore timing issues and dynamic aspects which allows us to concentrate on the basic trade-offs in the most elementary framework. We also formulate the model in terms of changes relative to a given "Business As Usual" scenario, which is defined as the situation without geoengineering and without abatement; the sum of CO_2 concentrations and associated absorption capacity in this scenario, $\bar{S}^{LF}$, is taken as a parameter.

1.3.1 Optimal Climate Change Policy

We now study how geoengineering affects optimal climate change policy, i.e. the policy that minimizes the sum of damages and economic costs by choosing the level of CO_2 concentrations, S, the amount of abatement, A, and the level of geoengineering,

G. This policy follows from the following minimization program:

$$\min_{\{A,G,S\}} C(A) + M(G) + D(S,G) \text{ subject to (1.3).} \tag{1.6}$$

We proceed in two steps. First we keep G constant and determine the optimal CO_2 level $s(G)$ with associated abatement policy $a(G) = \bar{S}^{BAU} - s(G) - \delta(s(G), G)$. Second, we keep S fixed and determine the optimal geoengineering policy $g(S)$. Finally, we combine the two to study the optimal mix of abatement and geoengineering.

Exogenous geoengineering

The first corder conditions with respect to A and S (for given G) associated to the minimization problem in (1.6), can be written as (see appendix for details):

$$\tau = C_A\left(\tilde{a}(S,G)\right) \tag{1.7}$$

$$\tau = \frac{D_S(S,G)}{1 + \delta_S(S,G)} \tag{1.8}$$

where $\tilde{a}(S,G) \equiv \bar{S}^{BAU} - S - \delta(S,G)$ is the level of abatement needed to satisfy condition (1.3), and τ is the Lagrange multiplier associated to the carbon cycle constraint (1.3). As usual, τ has the interpretation of the social marginal cost of carbon or the pollution tax to which firms equate the marginal abatement cost, as in equation (1.7). In the optimum, the pollution tax has to be equal to the marginal damages net of the contribution of increased CO_2 concentration to the absorption capacity, as in equation (1.8).

Equations (1.7) and (1.8) determine the optimal CO_2 concentration, S, and pollution tax, τ, for a given level of geoengineering, G. In particular, from (1.7) and (1.8) we find that the optimal S is determined by

$$C_A = \frac{D_S}{1+\delta_S}, \tag{1.9}$$

This is an implicit function in S and G, which we can solve as $S = s(G)$.

We are interested in analyzing the effects of the introduction of geoengineering in the economy. We study the experiment in which we move from a non-geoengineering scenario to an scenario in which we exogenously introduce geoengineering at a level $\hat{G} > 0$, that we assume to be small. We report the results from this experiment in our first proposition.

Proposition 1: *Suppose an interior optimal choice of abatement and CO_2 concentration exist, then, in comparison to the optimum without geoengineering, the (marginal) introduction of geoengineering induces*

(i) less abatement and a lower tax in the optimum, and

(ii) higher (lower) CO_2 if the following inequality, evaluated at the optimum without geoengineering, holds (is violated):

$$\frac{\delta_{SG}C_A - D_{SG}}{(1+\delta_S)C_{AA}} > \delta_G \tag{1.10}$$

Proof is in the Appendix

To illustrate these results we can use a standard diagram in which marginal damages and marginal abatement costs are depicted as a function of the level of pollution. In particular, we depict equations (1.7) and (1.8) in the S vs. τ plane as the MAC and MD curve respectively.

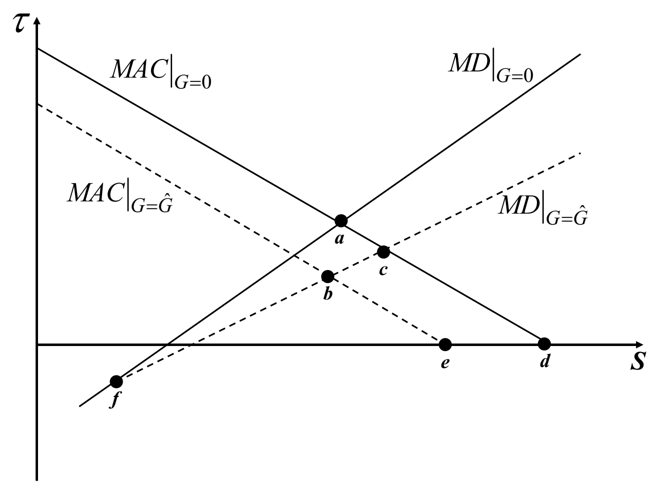

Figure 1

Initially the economy is in an equilibrium where there is no geoengineering intervention (point a). Geoengineering shifts the marginal damage cost curve down, since the cooling effect of geoengineering reduces marginal damages. It also shifts the marginal abatement cost curve down: the cooling effect of geoengineering increases carbon uptake and implies that less abatement costs have to be incurred to neutralize emissions. This implies that absorption (δ), which acts as "natural abatement", has to be substituted away from "human-made abatement" (A). With both curves shifting, the overall impact is that abatement and the optimal tax unambiguously decreases, but the concentration of CO_2, S, may either rise or fall, depending on whether the effect on absorption is weak or strong, respectively.

Geoengineering reduces the need to tax emissions for three reasons. The cooling effect of geoengineering makes increases in temperature through CO_2 accumulation less harmful, first through lowering marginal damages ($D_{SG} < 0$) and, second, through increasing the marginal rate of carbon absorption ($\delta_{SG} > 0$). Third, the cooling effect of geoengineering increases total carbon uptake through diminished heterotrophic respiration ($\delta_G > 0$). All these effects lower the social costs of carbon and result in an optimal lower carbon tax. With lower carbon taxes, traditional mitigation measures are less needed.

The effect of geoengineering on CO_2 concentration is ambiguous. On the one hand, the lower marginal damage induces lower abatement. On the other hand, improved carbon uptake from the atmosphere decreases CO_2 concentration in the atmosphere. Indeed, according to the inequality in (1.10), if the change in carbon uptake as a result of geoengineering (δ_G) is small, the first effect dominates and geoengineering makes it optimal to allow for higher concentrations of CO_2 than without geoengineering.

The optimal level of geoengineering for given abatement

We now confront costs and benefits of geoengineering to discuss its optimal level. Referring to figure 1, we find that the net benefits of the introduction of a small amount of geoengineering are given by the area abf minus the area *bcde*. If this is a positive amount, geoengineering is optimal to introduce at some level. By varying the level G, net benefits will change. To determine the level of geoengineering for which net benefits are maximized, we first determine the level of geoengineering that minimizes total social cost in (1.6) for given abatement level A.

The first order condition in problem (1.6) for optimal choice of geoengineering activities G is given by:

$$\delta_G D_S/(1+\delta_S) - D_G = M_G \tag{1.11}$$

The left-hand side of this equation represents the marginal benefits from geoengineering: cooling reduces damages ($D_G < 0$) and increases the absorption of carbon ($\delta_G > 0$), which has a marginal social cost $D_S/(1+\delta_S)$. The right-hand side of the equation represents the marginal costs of geoengineering, ($M_G > 0$). Equation (1.11) defines an implicit function with G and S as the only variables. Invoking Lemma 1, we use the implicit function theorem to express G as an increasing function of S (see appendix). Hence, we write the following lemma:

Lemma 3: *If (1.6) has an interior solution, then for given A, social cost is minimized for $G = g(S)$; where $G = g(S)$ solves (1.11) with: (i) $g'(S) > 0$ and (ii) $g(S^{\bar{T}}) = 0$.*

Proof: in the Appendix

The function $g(S)$ gives the optimal choice of geoengineering for a given level of abatement, but expressed as a function of atmospheric CO_2 concentration.[3] We refer to this equation as the optimal supply of geoengineering. More CO_2 in the atmosphere makes it optimal to supply more geoengineering. An increase in the concentration of CO_2 raises the benefits of geoengineering since CO_2-induced higher temperature is associated with higher marginal damages, which geoengineering can offset. Hence, more CO_2 justifies more geoengineering.

Geoengineering and abatement optimized

We can now substitute $G = g(S)$ into (1.7) and (1.8), and depict the resulting expressions in the (S, τ) plane, as in Figure 2. Point b in the figure represents the

[3] To see why it is the optimal response to given A rather than given S, note that the first order condition for abatement, (1.7), is not included in (1.11), but the one for S, (1.8), is included. The optimal response of G to given S would be given by $-D_G = M_G - \delta_G C_A$. We choose to take A as given since this corresponds most easily to our model with two types of players, see below.

optimum choice of S and τ with optimized geoengineering and abatement, and can be compared to the optimum without geoengineering, point a.

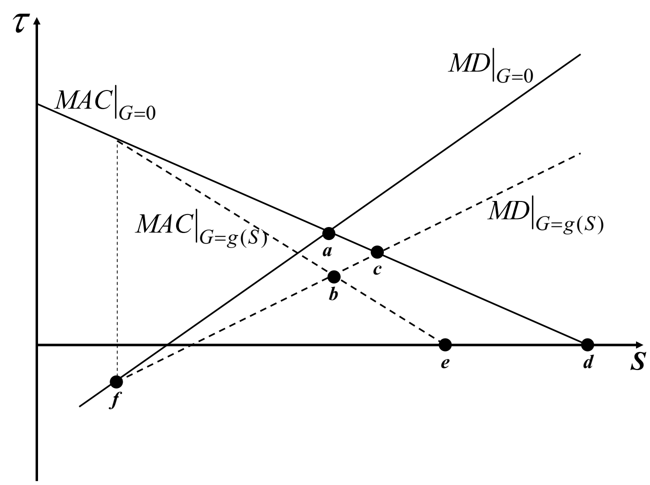

Figure 2

In Figure 2 we have depicted a situation in which, when geoengineering is optimized, the optimal tax level is greater than zero, which implies a positive level of abatement. Geoengineering does not make abatement redundant. As is clear from (1.7) and (1.8), a positive social cost of carbon, τ, requires that $C_A(a(S, g(S)))$ and $D_S(S, g(S))$ are positive for values of S in the neighborhood of the point where $C_A = D_S/(1 + \delta_S)$ (recall $\delta_S > 0$).

With optimal geoengineering, a zero or negative social cost of carbon can no longer be ruled out theoretically. Geoengineering reduces marginal damages and marginal abatement costs (C_A and D_S, respectively). If these reductions are big enough, the social cost of carbon becomes negative. Formally, for this to happen, we need $C_A < 0$

and $D_S < 0$ close enough to (and at) the point in which $C_A = D_S/(1+\delta_S)$. The former, negative marginal abatement costs, implies that society should emit more than would be profitable in the Business As Usual situation; the latter, negative marginal damages, implies that society benefits from higher CO_2 stocks, which is possible when the fertilization effect dominates the geoengineering-mitigated temperature effect. We formalize this in the following proposition:

Proposition 2: *Consider the optimum with geoengineering. Let* τ^* *denote the social marginal cost of carbon in this optimum;* S^{ZC} *denote the level of* S *for which marginal abatement costs are zero, i.e. the level that solves* $S^{LF} - S = \delta(S, g(S))$*; and* S^{ZD} *denote the level of* S *for which marginal damages are zero, i.e. the level that solves* $0 = D_S(S, g(S))$*. Then,*

(i) $\tau^* > 0$ *if and only if* $S^{ZD} < S^{ZC}$*;*

(ii) $\max\{S^{\bar{T}}, \bar{S}\} < S^{ZC}$ *is a sufficient condition for* $\tau^* > 0$*;*

(iii) optimal abatement is strictly positive if and only if $\tau^* > 0$*.*

Proof is in the Appendix.

The possibility of a non-positive tax can no longer be ruled out, since geoengineering can offset the damage from global warming: after stabilizing temperature (at $\bar{T}$ so that $d_2' = 0$), increases in CO_2 concentration improve welfare (i.e. reduce damages) through the fertilization effect ($d_1'(S) < 0$ provided $S < \bar{S}$).

The proposition only shows that a negative tax is possible. The real surprise is that with optimized geoengineering the cost of carbon can still be positive. Indeed, since geoengineering is subject to increasing marginal costs, stabilizing temperature for a large CO_2 level might be too costly. Moreover, for a large stock of CO_2, acidification problems may dominate the fertilization effect.

1.4 Example and Calibration

To illustrate the results in Proposition 2, and in order to make them more explicit, we approximate all our functions as quadratic polynomials and solve explicitly for the social marginal costs of CO_2. In particular damages are equal to:

$$d_1(S) = \eta_1(S^2/2 - \bar{S}S),\ d_2(T) = \eta_2(T^2/2 - \bar{T}T),\ d_3(G) = \eta_3 G^2/2 \tag{1.12}$$

while abatement and geoengineering costs are given by:

$$C(A) = \alpha A^2/2,\ M(G) = \gamma G^2/2 \tag{1.13}$$

We also linearize the temperature function around the preindustrial concentration of CO_2.

$$T(S,G) = \overline{\beta} S - \lambda G \tag{1.14}$$

where $\bar{\beta} = \lambda\beta/S_0$, $\beta = 5.35$ Wm^{-2}.

We further simplify and assume that the absorption capacity of the atmosphere is linear in S and it is not affected by geoengineering.

$$\delta(S,G) = \bar{\delta}(S - S_0) \tag{1.15}$$

This assumption transforms the carbon cycle equation into a linear relation between CO_2 concentrations and abatement. Below, we analyze the sensibility of the results to this simplifying assumption by running numerical simulations for different values of $\bar{\delta}$.

The chosen specifications imply the following closed-form solutions to the optimal levels of the endogenous variables:

$$S^* = \frac{\psi S^{ZC} + S^{ZD}}{1+\psi},\ G^* = \bar{\beta}\zeta S^* - \zeta\bar{T},\ \text{and}\ \tau^* = \alpha A^* = \frac{\alpha(1+\bar{\delta})}{1+\psi}(S^{ZC} - S^{ZD}).$$

where

$$S^{ZC} = \frac{\bar{S}^{BAU} + \bar{\delta}S_0}{1 + \bar{\delta}} \text{ and } S^{ZD} = \phi\bar{S} + \varphi\bar{T}.$$

with

$$\phi = \frac{\eta_1(\gamma + \eta_2\lambda^2 + \eta_3)}{(\gamma + \eta_2\lambda^2 + \eta_3)\eta_1 + (\gamma + \eta_3)\bar{\beta}^2\eta_2}, \varphi = \frac{\eta_2\bar{\beta}(\gamma + \eta_3)}{(\gamma + \eta_2\lambda^2 + \eta_3)\eta_1 + (\gamma + \eta_3)\bar{\beta}^2\eta_2},$$

$$\psi = \frac{\alpha(1 + \bar{\delta})^2(\gamma + \eta_2\lambda^2 + \eta_3)}{(\gamma + \eta_2\lambda^2 + \eta_3)\eta_1 + (\gamma + \eta_3)\bar{\beta}^2\eta_2}, \text{ and } \zeta = \frac{\eta_2\theta}{(\gamma + \eta_2\lambda^2 + \eta_3)}.$$

Looking the result for the optimal tax τ^*, we can see that the it is positive and abatement is needed in the optimum if and only if:

$$S^{ZC} > S^{ZD}$$

which for this specific assumptions yield:

$$\tau^* > 0 \text{ if and only if } \bar{S} < \bar{\bar{S}}(\bar{\delta}, \chi)$$

where

$$\bar{\bar{S}}(\bar{\delta}, \chi) = \frac{1}{\phi}(S^{ZC} - \varphi\bar{T})$$

How large $\bar{\bar{S}}$ is, relative to S^{BAU}, depends on two main unknown parameters. First, it depends on the costs of geoengineering $\chi = \gamma + \eta_3$. If the costs of geoengineering are low, temperature effects can be reduced at a very low cost, and it would be easier for the fertilization effect to dominate. Second, when the absorption capacity of the biosphere is large, that is $\bar{\delta}$ is large, the magnitude of $\bar{\bar{S}}$ is reduced which makes the possibility of a negative tax more likely.

Using very standard results in the economics of climate change (Nordhaus 2008), we calibrate our model to the year 2100. For this year the BAU concentrations of CO_2 are in the order of $S^{BAU} = 685$ppm. The radiative forcing at that concentration level is given by $R^{BAU} = 5.35\ln(S^{BAU}/S_0)$ with $S_0 = 270$ppm. Associated to this radiative forcing there is a change in temperature given by $T^{BAU} = 4.28^o$K.

If no action is taken to deal with climate change, Nordhaus (2008) finds that around 3% of global GDP (around \$50 trillion per year) will be lost in the process. A resent study by Brander et.al. (2009) shows that ocean acidification damages will add 10% to the damages caused by temperature. Using these results and assuming $\bar{S} = 0$ and $\bar{T} = 0$, we can calculate the values of the parameters η_1 and η_2. In particular we find that:

$$\eta_1 = 1.65 \times 10^8 [\$/ppm] \text{ and } \eta_2 = 1.55 \times 10^{13} [\$/^oC]$$

It has been estimated that the cost of reducing concentration levels back to preindustrial levels will require an economic effort in the same order of magnitude that the damages if nothing is done (Nordhaus 2008, Goulder and Mathai 2000). We assume that the costs of abatement back to preindustrial levels are 3% of global GDP. The cost of abatement per ppm is equal to:

$$\alpha = 1.65 \times 10^9 [\$/ppm]$$

The lower the value of $\bar{\bar{S}}(\bar{\delta}, \chi)$, the more likely the possibility of $\tau^* < 0$. The minimum value of $\bar{\bar{S}}$ is obtain when $\bar{\delta} = 1$ and $\chi = 0$. For those conditions we can see that $\bar{\bar{S}} \in [477.5\text{ppm}, 685\text{ppm}]$. That is, even for the unlikely case that geoengineering damages are small and absorption is large ($\bar{\delta} = 1$ implies no CO_2 accumulation), $\bar{S}$ has to be really close to S^{BAU} before taxes become negative. That is, when CO_2 related damages, like precipitation changes or ocean acidification, are not a problem only until CO_2 concentrations becomes really high.

The figure below shows a contour plot of $\bar{\bar{S}}$ for $\bar{\delta} \in [0, 1]$ and $\chi \in [0, 2.5 \times 10^{11}]$. The shaded area shows he combination of values for which $\bar{\bar{S}} < 685$. The white area means $\bar{\bar{S}} > 685$. The contour lines show the values of $\bar{\bar{S}}$. The lighter the shade of the area, the lower is $\bar{\bar{S}}$; thus, the higher is the possibility of $\tau^* < 0$. The figure shows that $\bar{\bar{S}}$ is decreasing in $\bar{\delta}$ and increasing in χ.

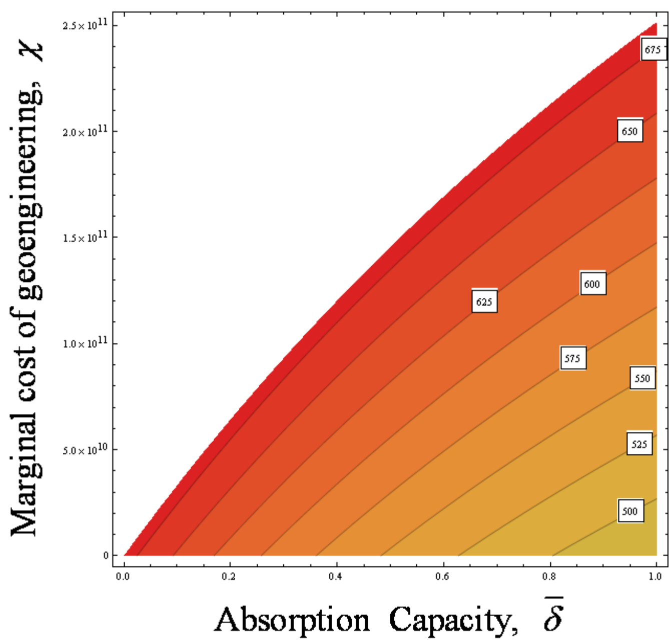

Figure 3

In order to have negative taxes three different conditions need to hold. First, costs of geoengineering must be low (that is including possible side effects). Second, the absorption capacity of the biosphere must be large. Finally, CO_2 related damages (e.g. precipitation changes and ocean acidification) must not become a problem only until very high levels of concentration of CO_2. The combination of these conditions make the possibility of a negative tax very slim; however it cannot be ruled out theoretically.

1.5 The risks of unilateral action

So far we studied the social optimum only. We now turn to a multi-country setting in which free-riding problems lead to second-best results.

Traditionally, countries spend too little on abatement since the benefits accrue partly to other countries while the country bears the full costs of its abatement. We will now show that the presence of geoengineering worsens this situation.

We consider the situation in which n symmetric countries, with $n >> 1$, choose abatement and one single "philanthropist" (a single country, or a single NGO or even a single private person) takes care of geoengineering. Each country i chooses its abatement level a_i to minimize its own costs $d(S,G)+c(a_i)$, subject to $a_i + \sum_{-i} a_j = \bar{S}^{BAU} - S - \delta(S,G)$, but takes the abatement of other as given. Geoengineering is also taken as given. Here $nd(S,G) = D(S,G)$ and $nc(A/n) = C(A)$ are world-wide damages and abatement costs, respectively, as modeled in the previous sections. Each country chooses abatement according to the following first-order condition:

$$C_A = \frac{1}{n}\frac{D_S}{1+\delta_S} \tag{1.16}$$

This result is standard: When carbon is a public bad (i.e. when $D_S > 0$), $A = na_i$ falls short of the optimal aggregate abatement level. This follows immediately from comparison of (1.16) with the optimal abatment rule (1.9). Countries free-ride when abating the public bad and thus overprovision of the public bad emerges. Similarly, carbon emissions are too low (abatement is not negative enough) when carbon is a public good (i.e. when $D_S < 0$): countries then free ride in the provision of the public good. Recall that the fertilization effect of carbon can turn carbon into a public good.

After substitution of (1.3), equation (1.16) is an implicit equation in S and G, and can be written as a reaction function $S = \hat{s}(G)$ which represents how the abatement choice of individual countries results in a carbon stock S given geoengineering G. In Figure 4, we plot the free riders' best response as the line labeled "Free rider". We also plot the socially optimum best-response curve $s(G)$ (the line labeled "coordination"), which is defined by (1.9) as derived above. Both best-response curves, $S = \hat{s}(G)$ and $S = s(G)$, are increasing, since the more cooling through geoengineering, the less damaging emissions are (see proposition 1(i) with respect to s; the proof with respect to $\hat{s}$ is similar). The two best-response curves intersect at the combination of S and G for which no abatement is needed (since at the intersection, we have $D_S = D_S/n$, which requires $D_S = 0$ and hence $C_A = 0$, which requires $A = 0$).

The philanthropist maximizes world welfare by choosing G, taking abatement as given. This results into the choice $G = g(S)$ which was defined above (lemma 3).

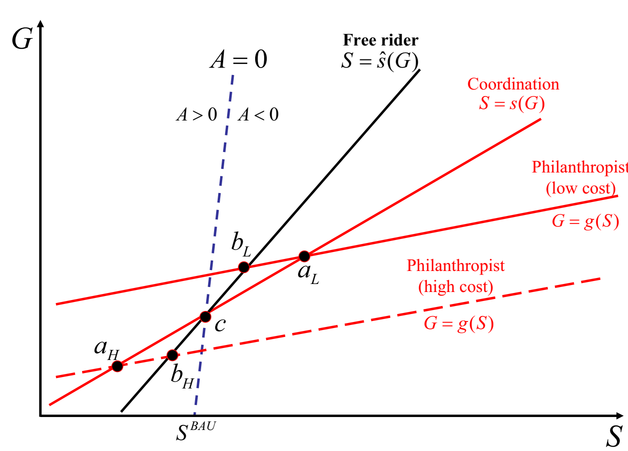

Figure 4

In Figure 4 we draw two possible $g(S)$ curves. The curves have a positive slope since $g' > 0$ (see lemma 3). The two curves arise for different costs of geoengineering: the broken curve is drawn for a higher cost of geoengineering than the solid curve.[4] The curves are drawn in such a way that the high cost results in carbon being a public bad (intersection between g and s to the left of $A = 0$), and the high cost results in carbon being a public good (intersection between g and s to the right of $A = 0$). From Figure 4, we can directly derive the following results:

Proposition 3: *In the uncoordinated equilibrium, with n countries free-riding when choosing abatement, and a single agent choosing geoengineering,*

(i) aggregate abatement is positive and sub-optimally low, while emissions, CO_2 concentration, and geoengineering activity are sub-optimally high, if the first-best tax is positive;

(ii) aggregate abatement is negative and sub-optimally high, while emissions, CO_2 concentration, and geoengineering activity are sub-optimally low, if the first-best tax is negative.

Proof: In text and graphically using Figure 4.

In Figure 4, points a_L and a_H are the optimal provision of abatement, while points b_L and b_H are the second-best levels of abatement. Part (ii) in Proposition 3 comes from observing that both CO_2 concentration and the geoengineering level evaluated at point a_L are higher than those evaluated at point b_L. Since point a_L is situated to the right of the $A = 0$ line, CO_2 is a public good: the social optimum requires an increase in emissions relative to the BAU situation (negative abatement).

[4]Formally, we assume $M(G) = \gamma m(G)$ where γ is a positive constant, and draw $g(S; \gamma)$ for two values of γ.

Part (i) in Proposition 3 is the reverse situation; and it is the standard result. CO_2 concentrations evaluated at point a_H are lower than those evaluated at point b_H. Given that points a_H and b_H at situated to the left of the $A = 0$ line, it implies that CO_2 is a public bad.

The first key insight is that geoengineering does not fully solve the coordination problem. It is optimal to undertake both abatement and geoengineering. When choosing the level of geoengineering, the philanthropist takes into account the global effects. But the geoengineering choice is conditional on the abatement level, which is the outcome of the decisions of a large group of countries who ignore global effects.

The second key insight is that geoengineering can turn the climate change problem from a coordination problem with overprovision of a public bad into one with underprovision of a public good.

1.6 Conclusions

Our model revisits the economics of climate change by separating temperature and GHG stocks. Geoengineering can affect the former but not the latter. Thus geoengineering is different from "cheap abatement": it offsets temperature-related damages, but it results into higher CO_2 concentrations if geoengineering is used as a substitute for traditional mitigation measures. As a result, even when geoengineering is cheap and has little harmful side-effects, geoengineering can never fully substitute for traditional mitigation measures that reduce the stock of CO_2. It turns out that geoengineering reduces abatement costs. Because cooling results in increased carbon uptake by the biosphere, less emissions reductions are needed to arrive at the same level of CO_2 concentration, as compared to the situation without geoengineering.

However, this effect is quantitatively likely to be small.

Geoengineering is unlikely to make fossil fuels harmless (or even benign). The reason is that CO_2-related damage occurs even when temperature is stabilized. Two examples to be known at current insights are precipitation changes and acidification of oceans, but many more examples might come to light.

Our results for the second-best economy shows that geoengineering worsens existing coordination problems in climate change policy. Moreover, if a single agent overestimates the net world benefits of geoengineering, it might introduce it at huge cost for the world economy.

Some might claim that it is simply to risky to interfere with the climate system at so big a scale. Our framework could serve to explore risk and other issues of geoengineering.

Chapter 2

Climate Policy under Uncertainty: A Case for Geoengineering

It appears to be technically feasible to engineer an increase in albedo, a planetary brightening, as a means to offset the warming caused by carbon dioxide (CO_2) and other greenhouse gases through Solar Radiation Management (SRM). This option has two characteristics that make it attractive for managing climate risk: it is quick and cheap. However, SRM cannot exactly compensate for the CO_2-driven climate change. Moreover, SRM introduces risks in the climate system that are unique to this type of intervention. We introduce SRM in a model of climate change economics and analyze the optimal policy under uncertainty. We find that the quick response allowed by SRM makes it important even if it is relatively ineffective at compensating for CO_2-driven climate change or even if its costs are expected to be large compared to traditional mitigation strategies. Finally, we examine the implications of risk on the effectiveness of SRM and show that the value of reducing this risk can readily exceed several trillion US dollars over the next 100 years, providing a strong argument for a research program.[1]

[1] This chapter is jointly written with David W. Keith

2.1 Introduction

It appears to be technically feasible to engineer an increase in albedo, a planetary brightening, as a means to offset the warming caused by carbon dioxide (CO_2) and other greenhouse gases through Solar Radiation Management (SRM) (Keith and Dowlatabadi 1992, Crutzen 2006, Blackstock et.al. 2009, Shepherd et.al. 2009). However, the cooling tendency produced by SRM does not exactly compensate for the change in CO_2-driven climate; and any particular method of SRM will no doubt entail other risks and side-effects, e.g. Bala et. al. (2008). Nevertheless, SRM may be a useful tool to mange climate risks (Wigley 2006). In this paper we use a stylized economic model to explore decision making under uncertainty regarding the effectiveness and the unintended consequences of SRM, as well as uncertainty regarding the magnitude of the climate damage posed by accumulating CO_2.

Uncertainty and inertia in the carbon-climate system are at the heart of our characterization of the long-term climate decision problem (van Vuuren 2008, Kallbekken and Rive 2007). Uncertainty is ubiquitous in the climate system; we know that warming will melt Arctic tundra which can then oxidize releasing carbon that causes more warming, but we know little about how large this effect may become (Global Carbon Project 2009). Similarly, plants grow faster in a carbon rich atmosphere, but it is unclear how much extra carbon will be absorbed by plants in a warming world (Long et. al. 2006).

Inertia refers to the long lags between the response of the climate system and the anthropogenic carbon emissions that preside climate change. The inertia of the carbon-climate system makes it impossible to quickly reduce climate risk by reducing emissions; up to 50% of the carbon accumulated today will remain in the atmosphere for the next 10,000 years (Solomon et. al. 2009).

Systems that combine high uncertainty and high inertia are difficult to control because we cannot predict their future behavior and we cannot quickly correct for undesired behavior. Suppose the response to carbon emissions of the climate system exceeds current median expectations. By the time we detect the strong climate response, it may be too late to manage the impacts by cutting emissions no mater how quickly they are eliminated (Matthews et. al. 2009, Caldeira and Matthews 2007, Taylor 2009).

Given that impacts are generally assumed to increase nonlinearly with the change in global temperature, and considering uncertainty in slow-responding elements of the system such as the cryosphere, it is possible that uncertainty about climate impacts spans an order of magnitude between the upper and lower 10% of a probability distribution (Roe and Baker 2007, Weitzman 2009). Moreover, this uncertainty is irreducible over a timescale of decades during which we will make near-term decisions about emissions mitigation. In 1979 the "Charney Report" by the US National Academy of Sciences provided an early and influential estimate of the uncertainty in estimating the climate response to CO_2. The report concluded that a doubling of CO_2 would increase the global average temperature between 1.5^{o}C and 4.5 ^{o}C. In spite of enormous improvements in scientific understanding of the climate system over the last three decades, we have not substantially narrowed the uncertainty about climate change due to the accumulation of greenhouse gases, which according to the IPCC 2007 is centered around 3^{o}C with little chance on being lower than 2^{o}C and higher than 4^{o}C (IPCC, 2007).

The term geoengineering is now applied to a wide range of technologies ranging from giant mirrors in the L1 point (Angel 2006) to ocean iron fertilization (Lampitt et. al. 2008). We distinguish two modes of geoengineering: SRM and Carbon Dioxide Removal (CDR). These modes differ in two principal ways. First, a reduction in the incoming radiation has almost instantaneous effects on global temperature; after Mount Pinatubo's explosion in 1991 global surface temperatures cooled about 0.5^{o}C over the following year (Soden et. al. 2002). CDR is necessarily slower, because the removal flux grows at the rate at which the capital stock of CDR is accumulated, and the reduction in CO_2 concentration grows as the integral of the flux. Thus, there is a long time delay between commitment to implement CDR and any significant reduction in climate impacts (Keith et. al. 2006). Second, SRM cannot exactly reverse the climate change caused by greenhouse gases. A reduction in insolation using SRM produces climate change that partially counter the climate change due to greenhouse gases. For example, if SRM is used to bring global average temperatures back to the preindustrial level it will tend to over control precipitation reducing it below preindustrial means and will do little to alter the impact of CO_2 on ocean acidification (Caldeira and Matthews 2007). Conversely, CDR can, ignoring hysteresis, fully correct CO_2 driven climate change, although any given CDR method may cause other environmental problems (e.g. see The Economist note "Who ate all the Algae", March 26, 2009).

In this paper we concentrate on SRM because this response mode differs most fundamentally from emission mitigation. Our analysis aims at a parsimonious representation of the following stylized facts: First, *SRM is imperfect* (Caldeira and Wickett 2003, Caldeira and Wood 2008, Robock et.al. 2008); in order to fully eliminate the effects of climate change we need a combination of abatement and SRM.

Second, *SRM is fast* (Govindasamy and Caldeira 2000; Govindasamy et al. 2002, 2003; Matthews and Caldeira 2007, Robock et.al. 2008); which implies that we can wait to observe the magnitude of the damage from climate change and then implement some level of SRM appropriately. This is the key fact that makes SRM a effective tool to manage climate risk. Third, *SRM is cheap* (NAS 1992, Blackstock et.al. 2009, Shepherd et.al. 2009); which makes SRM essential to achieve any temperature target at lower costs relative to traditional mitigation. Finally, the effectiveness and side effects of *SRM are uncertain*. The intentional modification of the earth's climate may lead to nonlinearities in the system creating even greater problems: changes in precipitation (Bala et. al. 2008), ozone depletion (Solomon et. al. 1996; Solomon 1999), and health issues (Crutzen 2006).

The advantages of SRM as a method to manage climate risks are twofold. First, it is inexpensive compared to mitigation, and second it allows rapid action avoiding some of the inertia of the carbon system. The corresponding disadvantages of SRM is that it imperfectly (or ineffectively) compensate for CO_2 driven warming and it may introduce unintended consequences. Our goal is to explore the trade-offs between these advantages and disadvantages in a simple decision analytic framework.

The rest of the paper proceeds as follows. In section 2 we define and calibrate the model. In section 3 we introduce uncertainty on the climate system and analyze the role of geoengineering in dealing with high-impact, low-probability outcomes. In section 4, we deal with the uncertainty attached to the implementation of geoengineering and analyze the value of reducing this uncertainty. We draw conclusions in section 5.

2.2 A General Description of the Model

2.2.1 Definitions and assumptions

When the concentration of greenhouse gases increase in the atmosphere it alters the balance between incoming solar radiation and outgoing terrestrial radiation, resulting on a increase in the mean global temperature of the Earth. Changes in mean global temperature, ΔT, are defined as a linear function of *radiative forcing*, R:

$$\Delta T = \lambda R \tag{2.1}$$

where λ is the climate sensitivity parameter. Radiative forcing is a function of the concentration of CO_2 in the atmosphere, S, relative to the preindustrial level (IPCC 2007):

$$R = \beta Ln(S/S_0) \tag{2.2}$$

where S_0 is the preindustrial level of CO_2 in the atmosphere. The parameter $\beta = 5.35$ Wm^{-2}.

When SRM is introduced in the model, the direct relation between CO_2 concentrations and temperature is altered. In particular, if we measure geoengineering in terms of its radiative forcing potential, we obtain an equation of the form:

$$\Delta T = \lambda(R - \varphi G) \tag{2.3}$$

where $\varphi \in [0, 1]$ captures the idea that geoengineering do not perfectly compensate for changes in mean global temperature caused by climate change (e.g. Heckendorn et.al. 2009). For example, when $\varphi = 1$, one watt per meter squared of geoengineering compensates for one watt per meter squared of CO_2-driven climate change. When

$\varphi = 0.5$, two units of geoengineering are needed to compensate for one unit of CO_2-driven climate change. Thus, for any given amount of effective radiative forcing reduced by geoengineering, the impacts form geoengineering will increase by $1/\varphi$. When $\varphi = 0$ we are in a situation in which geoengineering is not available in the economy; only traditional mitigation strategies, or *abatement* for short, can be used.

Abatement, which we denote by A, refers to measures that reduce the concentration level of CO_2 in the atmosphere. In particular, assume that $S = S^{BAU} - A$, where S^{BAU} is the business as usual concentration of CO_2 in the atmosphere.

The impacts of this change in temperature come in the form of increased precipitation, more unpredictable weather patterns and storms, sea-level rise, desertification and loss of fertile soils, etcetera. SRM technologies can intervene to restore the surface temperature by reducing the incoming solar radiation. Of course, this intervention cannot eliminate all the damages caused by climate change. In particular, the temperature compensation has a different regional distribution, leaving poles under compensated while the equator is over compensated (Matthews and Caldeira 2007). Moreover, the accumulation of greenhouse gases also have direct implications on the precipitation patterns (Allen and Ingram, 2002); and, in the case of CO_2, ocean acidification (Caldeira and Wickett, 2003, 2005).

Geoengineering also introduces damages on its own. Recent numerical simulations show that geoengineering will affect precipitation patterns and volumes, causing a decrease in precipitation over land and an increase in precipitation over the ocean (Bala et.al. 2009; Matthews and Caldeira, 2007; Trenberth and Dai, 2007); possibly causing droughts in large regions of the planet. Sulfuric acid deposition may create health and regional problems (Crutzen, 2006); although recent literature suggests these effects are small (Kravitz et.al., 2009).

We represent climate damages as the sum of three different sources of impacts: CO_2 concentration, changes in temperature, and geoengineering. We denote concentrations of CO_2 (measured in ppm) by S, ΔT are changes in temperature (measured in ^{o}C), and G captures geoengineering efforts (measured in W/m^2). Following Nordhaus 2008 and Goulder and Mathai 2000, we assume that damages are a quadratic function of their arguments. Thus total damages are given by

$$D(A,G) = \frac{1}{2}\eta_S(S^{BAU}-A)^2 + \frac{1}{2}\eta_T\lambda^2\left(\beta Ln\left(\frac{S^{BAU}-A}{S_0}\right) - \varphi G\right)^2 + \frac{1}{2}\eta_G G^2 \quad (2.4)$$

where η_S, η_T, and η_G represent the damages per unit of CO_2, temperature, and geoengineering, respectively. Notice that when A equals S^{BAU} and G equals zero, damages are zero. However, when A is less that S^{BAU}, damages are always positive. We capture the imperfections of geoengineering with two terms: First, φ captures the inefficiency of geoengineering at compensating for CO_2-driven climate change. Second, the term associated to η_G represents the damages associated to the unintended consequences of geoengineering.

We assume that abatement costs are increasing and convex. In particular, following Nordhaus (2008), we have:

$$\Lambda(A) = \frac{1}{2.8}K_A A^{2.8} \quad (2.5)$$

where K_A has units [\$ / ppm$^{2.8}$].

Following Keith and Dowlatabadi (1992) we assume that geoengineering costs are linear and given by

$$\Gamma(G) = K_G G \quad (2.6)$$

where K_G has units [\$(W/m^2)$^{-1}$].

Combining the definition in (2.4), (2.5) and (2.6), total costs of managing climate change can be written as:

$$\Omega = \underbrace{\frac{1}{2.8}K_A A^{2.8}}_{\Lambda(A)} + \underbrace{K_G G}_{\Gamma(G)} + \underbrace{\frac{1}{2}\eta_S(S^{BAU} - A)^2 + \frac{1}{2}\eta_T \lambda^2 \left(\beta Ln\left(\frac{S^{BAU} - A}{S_0}\right) - \varphi G\right)^2 + \frac{1}{2}\eta_G G^2}_{D(A,G)} \tag{2.7}$$

2.2.2 Calibration

We derive most of our calibration from the main results of the DICE 2007 model (Nordhaus, 2008). We set the year 2100 as the target year. For this year the business as usual concentration of CO_2 is in the order of $S_{2100} = 685$ ppm. The radiative forcing at that concentration level is given by $R_{2100} = 4.98$ W/m^2 with preindustrial concentrations equal to $S_0 = 270$ ppm. Associated with this radiative forcing there is an expected change in temperature given by $T_{2100} = 4.28^o$C.

If no action to deal with climate change is taken, around 3% of global GDP will be lost in 2100. We assume that damages caused by ocean acidification adds 10% to the total impacts from climate change (Brander et.al. 2009). Taking global GDP to be around $50 trillion per year, we can calculate the values of the parameters η_S and η_T:

$$\eta_S = 1.65 \times 10^8 \ [\$/\text{ppm}] \text{ and } \eta_T = 1.55 \times 10^{13} \ [\$/^o\text{C}]$$

According to the results of the DICE model, bringing the temperature change in 2100 down by 1^{o}C from its business as usual value will cost up to 1% of global GDP. Using this we can calculate the cost of abatement per unit of carbon (measured in ppm):

$$K_A = 1.65 \times 10^9 [\$/\text{ppm}]$$

Although at this stage it is very difficult to know the costs of geoengineering, some preliminary studies have suggested that SRM could offset the increase in global average temperature due to CO_2 at a cost 10 to 1000 times lower than achieving the same outcome by cutting emissions (Shepherd et. al. 2009, Blackstock et. al. 2009). In here, we assume that the costs of geoengineering are 1% of the costs of cutting emissions. Given the assumption of linear geoengineering costs, this allows us to estimate the unit costs of geoengineering:

$$K_G = 2.87 \times 10^{11} [\$(\text{W}/\text{m}^2)^{-1}]$$

There are two parameters yet to be calibrated, φ and η_G. However, there is not enough information to fully determine the value of these parameters, which is exactly the reason why geoengineering is considered to be highly risky. Thus, the analysis below is done for different values of φ and η_G. In this way, we capture the importance of these unknowns in determining the optimal level of geoengineering.

2.3 Geoengineering as Insurance

In this section we analyze the role of geoengineering in dealing with the uncertainty surrounding the climate's response to changes in the atmospheric concentration of CO_2. We introduce uncertainty in the climate sensitivity in the following way. We define the random variable $\widetilde{\lambda}$ that follows a two-point support distribution of the form:

$$\widetilde{\lambda} = \begin{cases} \lambda_H = 2.3 & \text{with probability } p = 1/10 \\ \lambda_L = 0.7 & \text{with probability } 1 - p = 9/10. \end{cases} \tag{2.8}$$

Notice that the mean of this distribution is $\hat{\lambda} = 0.86$, which is consistent with recent estimates (IPCC 2007). On the one hand, and because of the inertia of the carbon cycle, abatement needs to be chosen before uncertainty about the climate sensitivity is resolved. On the other hand, given the quick response of the climate system to geoengineering intervention, we can assume the choice of geoengineering is done after the uncertainty about the climate sensitivity has been resolved. Hence, we use a two-stage decision framework in which the abatement decisions are made in the first period and geoengineering decisions are made in the second period.

We analyze the optimal policy as a function of the expected marginal damages from geoengineering, $\eta_G \in [0, D_{CO_2}]$. We allow damages from geoengineering to be as high as those induced by CO_2-driven climate change; $D_{CO_2} = \$11.4 \times 10^{12}/(\mathrm{Wm}^{-2})$. That is, when $\eta_G = D_{CO_2}$, reducing temperature changes to zero using only geoengineering may create damages just as big as if temperature was equal to its business as usual level.

We consider three scenarios related to the effectiveness of geoengineering: $\varphi = 0.8$, $\varphi = 0.5$, and $\varphi = 0.2$; representing high, medium, and low effectiveness of geoengineering. Figure 1 shows the optimal abatement and geoengineering levels under the three different scenarios. As expected, geoengineering is a decreasing function of η_G while abatement is increasing in η_G. Also, comparing the three panels, note that abatement levels are higher when geoengineering is less effective. Thus abatement and geoengineering are *technical substitutes*: if geoengineering is costly or ineffective, it is optimal to implement more abatement. The three panels in Figure 1 also show that geoengineering is always higher in the bad outcome ($\lambda = \lambda_H$) compared to the good outcome ($\lambda = \lambda_L$).

The crucial result is that it is still optimal to implement high levels of geoengineering even if the marginal damages from geoengineering are as high as those of climate change ($\eta_G = D_{CO_2}$) or even if the effectiveness of geoengineering is very low ($\varphi = 0.2$). *The signal advantage of geoengineering is its quick response: the fact that it can be implemented after the uncertainty about climate sensitivity is resolved: the role of geoengineering as an insurance to be used in case of an unlucky climate outcome means that it is a valuable tool even if it is expensive or only partially effective.*

In the top panel of Figure 2 we compare the temperature change for the three different scenarios, and the temperature level in the absence of geoengineering. In the absence of geoengineering ($\varphi = 0$), the temperature change resulting from the implementation of the optimal policy is 2.4^{o}C. We can see that temperature change increases when the unit damages from geoengineering, η_G, increase. However, temperature never increases beyond its level in the absence of geoengineering ($\varphi = 0$). The increase in temperature occurs because there is a reduction in the level of geoengineering that is less than compensated by the increase in abatement levels; which comes from the fact that abatement costs are increasing and convex. This figure also shows that when geoengineering is more ineffective, temperature rises.

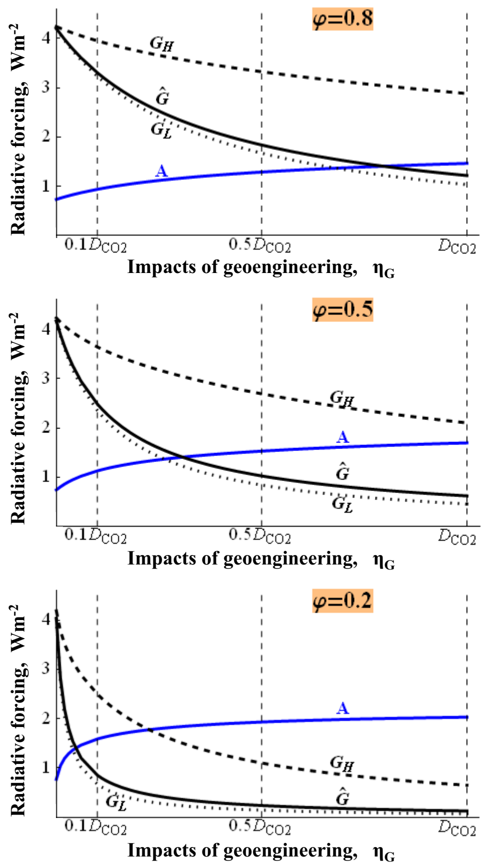

Figure 1. Optimal abatement and geoengineering levels as function of the impacts of geoengineering. The horizontal axis is the impacts of geoengineering expressed as a fraction of the business-as-usual climate damages. For example, when $\eta_G = 0.1D_{CO_2}$, the impacts of geoengineering are equivalent to 10% of the damages from CO_2-driven climate change. The vertical axis is in units of radiative forcing (Wm^{-2}). In each panel, A is abatement, G_L is the level of geoengineering if the climate sensitivity is low ($\lambda = 0.7$), G_H is the level of geoengineering if the climate sensitivity is high ($\lambda = 2.3$), and finally $\hat{G}$ is the expected value of geoengineering when the probability of $\lambda = \lambda_H$ is equal to $p = 0.1$. The top panel shows the results when the effectiveness of geoengineering is high ($\varphi = 0.8$), the middle panel shows the results when the effectiveness is moderate ($\varphi = 0.5$), and the bottom panel shows the results when the effectiveness of geoengineering is low ($\varphi = 0.2$).

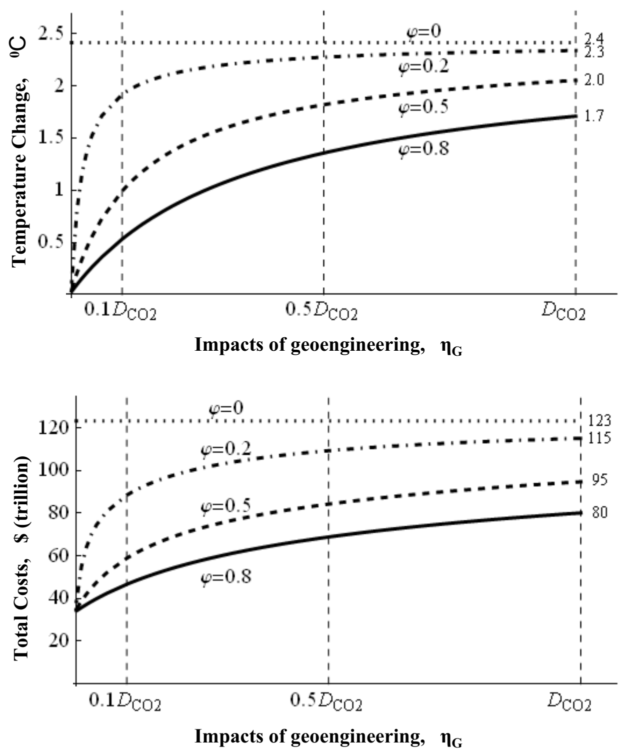

Figure 2. Temperature change and total cost evaluated at the optimal levels of abatement and geoengineering. The horizontal axis is the same as in Figure 1. The top panel presents the temperature change measured in ^{o}C. The results are shown for different effectiveness values. When $\varphi = 0$ geoengineering is not available. The temperature change evaluated at the optimal level of abatement in the absence of geoengineering is 2.4^{o}C. The bottom panel shows the costs of implementing the optimal policy. When $\varphi = 0$, total costs are equal to \$123 US trillion. Costs are calculated over the next 100 years.

The bottom panel in Figure 2 shows the total costs of managing climate change relative to the costs in the absence of geoengineering, for different values of φ, and as a function of the marginal damages from geoengineering , η_G. We can see that the more effective geoengineering is ($\varphi = 0.8$) the lower the costs of managing climate change relative to the case without geoengineering ($\varphi = 0$). As expected, the reduction in the total costs becomes less important when damages from geoengineering become larger. If geoengineering were to be harmless, $\eta_G = 0$, the savings relative to the case of no geoengineering would be around US \$89 trillion, equivalent to a reduction of around 73% of the expected costs of climate change. If on the other hand, $\eta_G = D_{CO_2}$ and $\varphi = 0.2$ the cost reduction due to the introduction of geoengineering is still worth US \$8 trillion. We reiterate here that *even if damages from geoengineering are substantially high, and its effectiveness is very low, it is still very valuable to have geoengineering available in the case the climate sensitivity is high, as a complement to abatement measures.*

2.4 Geoengineering Risk

The previous section discussed the benefits associated to the flexibility introduced by the possibility to choose geoengineering after the climate uncertainty has been revealed. In this section we expand our analysis to allow for risk on the effectiveness of geoengineering; we analyze the effects on the optimal policy and estimate the value of reducing this risk.

We introduce uncertainty on the effectiveness of geoengineering by defining a random variable $\widetilde{\varphi}$ that follows the distribution:

$$\widetilde{\varphi} = \begin{cases} \varphi_H = 0.8 & \text{with probability } q = 1/2 \\ \varphi_L = 0.2 & \text{with probability } 1 - q = 1/2. \end{cases} \tag{2.9}$$

which has an expected value of $\hat{\varphi} = 0.5$. The uncertainty about the distribution of geoengineering affects the way in which abatement and geoengineering interact. In particular, we can compare the results we obtain in the previous section, where the effectiveness in known as equal to $\varphi = 0.5$, to the results where the effectiveness is unknown, but its expected value equals $\hat{\varphi} = 0.5$.

Note that the distribution defined in (2.9) is a mean preserving spread of a the certainty case where $\varphi = 0.5$. As such, what we are reporting in Figure 3 can be understood as the effects of a riskier geoengineering intervention. In the top panel we can see that the introduction of risk in the effectiveness of geoengineering reduces the level of geoengineering and increases the level of abatement. This implies that the implementation of abatement works as an insurance against a bad geoengineering outcome; that is, abatement and geoengineering are *risk complements*. We explore this relation in more detail later in the text. The middle and bottom panels show that an increase in risk comes with higher temperatures and higher social costs for society. The effects of this type of uncertainty are limited by extend of the unintended consequences of geoengineering. The higher is η_G, the lower is the level of geoengineering and the lower are the effects of uncertainty on the optimal policy. In fact, if $\eta_G = 0$, uncertainty represents an increase of 0.83^oC relative to the certainty case, and the increase in the costs of climate change would be \$28 trillion. On the other hand, if $\eta_G = D_{CO_2}$, riskier geoengineering is equivalent to a world almost 0.1^oC warmer and the costs of climate change would be \$5 trillion higher.

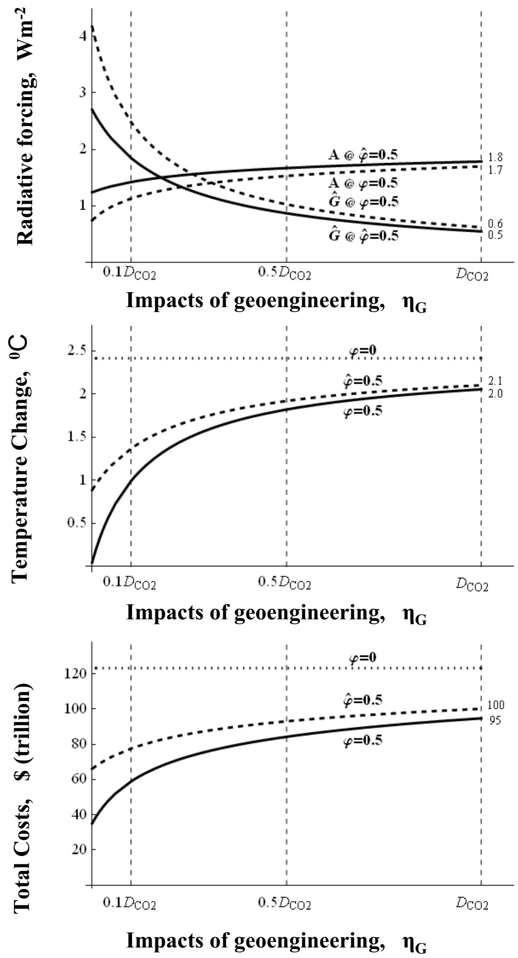

Figure 3. Optimal use of geoengineering and abatement when the effectiveness of geoengineering is uncertain. The top panel shows amount of radiative forcing due to abatement and geoengineering. In each case the horizontal axis is the impact of geoengineering (see Figure 1). The middle panel shows the effects of geoengineering risk on temperature change, measured in ^{o}C. The bottom panel shows the effects of geoengineering risk on the total costs for society. The dashed lines present the case with uncertainty on the effectiveness of geoengineering, $\tilde{\varphi}$ with $\hat{\varphi} = 0.5$, and the solid lines represent the case of no uncertainty, $\varphi = 0.5$.

2.4.1 The Value of Learning

Risk on the effectiveness of geoengineering can be reduced by research and small scale implementation of geoengineering[2]. How much is this research worth? To answer this question we define three distinctive problems in which learning occurs at different stages of the decision making process.[3] In the first problem we assume that learning occurs after abatement and geoengineering are chosen; we refer to this as *full uncertainty — FU*. In the second problem we assume that learning occurs before geoengineering decisions are made, but after abatement is chosen; we refer to this as *partial learning — PL*. In the third problem, we assume that learning occurs before abatement and geoengineering decisions are made; we refer to this as *early learning — EL*.

The left column in Figure 4 shows the effects of learning on the optimal policy (top panel) and the net savings or *expected value of perfect information* (bottom panel). The top panel shows that the earlier we learn about the effectiveness of geoengineering, the higher is the level of geoengineering and the lower is the level of abatement. In the bottom panel it is shown that partial learning generates savings of \$8 trillion when $\eta_G = 0.1 D_{CO_2}$ and \$1 trillion when $\eta_G = D_{CO_2}$ over the next 100 years. Early learning is worth \$10 trillion when $\eta_G = 0.1 D_{CO_2}$ and \$2.5 trillion when $\eta_G = D_{CO_2}$.

In order to expand on the idea of abatement and geoengineering being risk complements, we change the distribution over the effectiveness of geoengineering and define

[2]Small scale implementation also entails risks. Under the current specification we capture these damages in η_G

[3]We refer to learning here as the absolute resolution of the uncertainty surrounding geoengineering.

it in the following way:

$$\widetilde{\varphi} = \begin{cases} 0.5 + \sigma & \text{with probability } q = 1/2 \\ 0.5 - \sigma & \text{with probability } 1 - q = 1/2. \end{cases} \tag{2.10}$$

were σ is the standard deviation of this distribution and it is our measure of risk. In particular, by increasing σ we make mean preserving spreads of the distribution which allows us to analyze the results only in terms of increase risk.[4] We allow σ to vary between 0 and 0.3. When $\sigma = 0.3$, φ_L takes the value 0.2 and φ_H takes the value 0.8, which coincides with the original distribution given in equation (2.9).

For simplicity, and only for this exercise, we assume that the marginal damages from geoengineering are fixed and equal to $0.1D_{CO2}$. We show the results in the right column of Figure 4. We capture the effects of learning on the optimal policy in the top panel, and the net savings or *expected value of perfect information* in the bottom panel. In the top panel we can see that an increase in risk reduces the level of geoengineering and increases the level of abatement, reinforcing the idea that geoengineering and abatement are risk complements. Learning reduces risk, thus the earlier the learning, the more geoengineering there is, and the lower is the level of abatement. As it is shown in the bottom panel of the right-hand column, the expected value of perfect information increases with risk, and the difference between partial learning and early learning is $2.73 trillion when $\sigma = 0.3$. This result provides a clear argument for early research on geoengineering.

[4]Increases in risk as defined by Rothschild and Stiglitz 1970 and 1971.

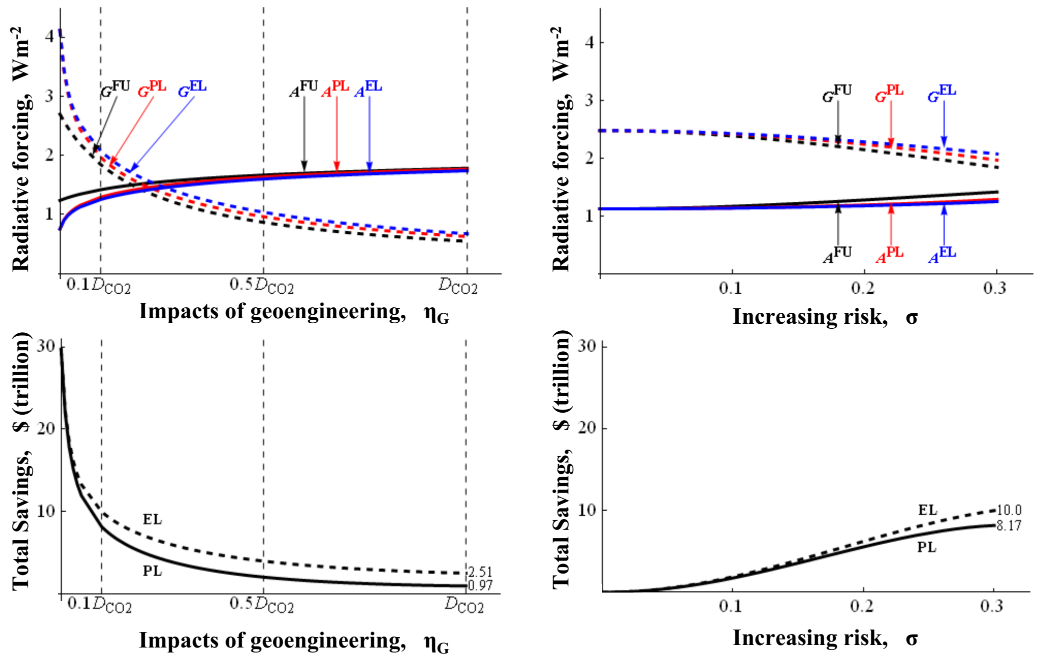

Figure 4. The effects of learning on the optimal levels of abatement and geoengineering, and its implications on temperature change and total costs. The left column shows the results in terms of the expected damages from geoengineering, η_G, while keeping $\sigma = 0.3$. The right column shows the results in terms of increasing risk of the distribution, σ, while keeping $\eta_G = 0.1D_{CO_2}$. The learning scenarios are denoted by FU (full uncertainty), PL (partial learning), and EL (early learning). The top row shows the effects of learning on the expected level of abatement and the expected level of geoengineering. The bottom row shows the total savings. Total savings are the difference between the total costs of the optimal policy when there is no learning and the corresponding learning scenario. Total savings in the bottom-left panel evaluated at $0.1D_{CO_2}$ are equivalent to the total savings in the bottom-right panel when $\sigma = 0.3$.

2.5 Conclusions

In this paper we developed a model of decision making under uncertainty in which a central planner chooses the level of abatement and geoengineering that minimizes the costs of climate change. Using this framework we draw three conclusions.

First, imperfect geoengineering is an effective means to approach the uncertainty in the climate response because it can be implemented after this uncertainty is resolved, providing a tool to manage the inertia in the carbon-climate decision problem. Without geoengineering, the high-consequence low-probability climate impacts drive very high levels of abatement due to the irreversibility of emissions. In our simplified model, geoengineering is used in the case of an unlucky (high-impact) outcome even if the damages from geoengineering exceed the damages from global warming, and even if geoengineering is not very effective. Under similar assumptions, geoengineering use is substantially reduced when climate impacts are relatively low.

Second, when we introduce uncertainty on the geoengineering option, abatement becomes a tool for managing this new risk. In other words, increasing risk in geoengineering causes an increase in the level of abatement in order to rely less on geoengineering. The combination of the the first two conclusions implies that abatement and geoengineering are complements in managing climate risks.

Third, we find that learning about geoengineering — that is the value of information associated with reducing the uncertainty about the effectiveness and side effects of geoengineering — may be worth several trillion dollars over the next 100 years.

This specific numerical result depends, of course, on the calibration of the model and on the assumptions about the prior probability distribution over the effectiveness of geoengineering in managing climate impacts. We have used a calibration of the economics of climate damages and abatement that is widely used and is representative of results derived in many complex models. In addition, we have assumed the expected damages from geoengineering greatly exceed current estimates; we therefore expect that the general result that the value of significantly reducing the uncertainty about geoengineering exceeds several trillion dollars is robust.

The model is of course, a very simple representation of the problem. The limitations of the model are the same attached to any model of a representative consumer; no strategic interaction, no asymmetries and therefore, no distributional issues. Also, the numerical results are attached to the mean preserving spread experiment, not to the specific distribution we choose.

We conclude by reiterating that what makes geoengineering vital for managing climate risk is not its cost but the fact that it can be implemented quickly, in the event of a "climate emergency". Research to reduce uncertainties about SRM technologies should therefore be a priority for the international scientific and decision making communities.

Chapter 3

The Long and Short of Climate Change: Abatement versus Geoengineering

Recent scientific advances have introduced the possibility of geoengineering the climate system to lower ambient temperatures without lowering greenhouse gas concentrations. This possibility has created an intense debate given the ethical, moral and scientific questions it raises. This paper examines the economic issues introduced when geoengineering becomes available in a standard two country model where strategic interaction leads to suboptimal abatement. Geoengineering naturally introduces the possibility of technical substitution away from abatement, but also affects the strategic interaction across countries. When countries are identical, I find these strategic effects create greater incentives for free riding, but if the effects of climate change are asymmetric, the prospect of geoengineering can induce greater abatement levels by some countries.[1]

[1]I want to thank Scott Taylor for his insights throughout this project.

3.1 Introduction

Climate change results from the accumulation of greenhouse gases (GHG) in the atmosphere. Until now, the international community has resorted to abatement strategies to deal with the warming caused by climate change; however, it is well understood that abatement is a global public good and as such, it suffers from under-provision due to free riding. Considering the inability of nations to coordinate effectively on the climate change problem, scientists are exploring new technologies designed to quickly lower temperatures without lowering GHG concentrations.[2] These technologies fall under the category of *geoengineering*. In this paper, I examine the economic issues introduced when geoengineering is made available in an world where strategic interaction leads to suboptimal abatement.

Specifically, I ask three questions: 1) does the presence of geoengineering increase the familiar free riding effect on abatement? 2) could the costs associated with this increase in free riding overwhelm the benefit gained from introducing a second instrument to deal with climate change? and 3) how does the presence of geoengineering change the geopolitical distribution of winners and losers from climate change policy?

I use a very conventional model to answer these questions. I consider a two-country partial equilibrium model where each country minimizes its own costs of managing climate change. The model has two key features. First, abatement and geoengineering are both global public goods. Second, the two countries interact in a two-stage strategic environment in which abatement decisions are made in the first stage and geoengineering decisions are made in the second stage. This timing of events

[2] A quick reduction in temperatures may be needed in the case of rapid or catastrophic climate change. For more on this topic see Taylor (2009)

arises naturally from the physical characteristics of abatement and geoengineering strategies, and the way they interact with the climate system. This timing also captures the idea that, although not yet available, the prospect of geoengineering technologies affects abatement decisions today.

Within this framework, I decompose each country's best response to a change in the other country's level of abatement into a *free riding effect*, a *technical substitution effect*, and a *strategic effect*. Using this decomposition, I investigate whether the introduction of geoengineering in one country will lead to a reduction in the abatement efforts of the other country. The general intuition is that abatement and geoengineering are strategic substitutes, and as such, there should be less abatement in the presence of geoengineering. Because of this result, it has been suggested in previous literature that geoengineering should be banned. I show that the reduction in abatement is only one possibility. It is possible that the introduction of geoengineering may increase the equilibrium level of abatement; in this case, abatement is used by one country as an instrument to deter the implementation of geoengineering by the other country.

Next, I analyze the effects of geoengineering on the wellbeing of society. In particular, I assume damages from climate change are a function of changes in global temperature; which in turn are proportional to *radiative forcing*. Radiative forcing describes how the balance between incoming shortwave radiation (energy coming from the sun) and outgoing long wave radiation (energy leaving the Earth's atmosphere in the form of heat) is affected by human activity. By defining abatement and geoengineering in terms of their radiative forcing potential, I obtain a linear relation between abatement and geoengineering in the damage function. I further assume that costs and damages are quadratic, which allows me to find linear best response functions, and closed form solutions, in both stages of the game.

I study two examples. First, I concentrate on the case where the two countries are identical. Second, I analyze a case of asymmetric countries in which countries differ in the extend of damages they perceive from climate change as well as damages caused by the unintended consequences of geoengineering. To simplify the analysis I assume that one country uses abatement and the other uses geoengineering; but no country can use both technologies. When countries are identical I find that introducing geoengineering causes a net reduction in abatement; however, total costs of managing climate change are also reduced. When I allow for countries to differ in the type of technology each have available I find that, for large asymmetries in the damages from temperature or the damages from geoengineering, abatement could increase due to the introduction of geoengineering. Moreover, I show that the increase in abatement could be so large that the country implementing abatement would do so at levels even greater than those found in the first best solution in the absence of geoengineering. This last result suggests that the possibility of geoengineering being implemented may create greater consensus on the level of abatement between countries that are affected differently by climate change.

The results of this model follow from very standard methods. The cost minimizing set up with increasing and convex costs and damages is standard for the analysis of climate change policies (Goulder and Mathai, 2000). The sequential nature of the model resembles the problems of capacity building and competition on output (Brander and Spencer, 1983 and Dixit, 1986). These types of models have also been used before to study the outcomes of international environmental agreements

(Barrett, 1994 and Endres, 1997). I draw heavily from this literature to solve the current model.

Research in physics and the natural sciences has been oriented towards the analysis of geoengineering as a technology available just in case society fails to reach an agreement to reduce emissions (Crutzen, 2006; MacCracken, 2006). However, it has been shown that the global warming caused by anthropogenic emissions can be minimized by using geoengineering technologies, which suggests there is a role for geoengineering to achieve any given temperature target at a very low cost (Keith and Dowlatabadi, 1992; Keith, 2000, 2001; Wigley, 2006; Rasch et.al. 2008). Even though scientists agree that research on geoengineering should be at the forefront of the research community (AMS, 2009), due to the possible unintended consequences attached to the idea of altering the climate system as a whole, it has been suggested the actual implementation of geoengineering to be highly regulated or restricted (Barrett, 2008; Victor 2008; Victor et.al., 2009; Weitzman, 2009).

The possibility of a less costly set of technologies to deal with climate change in the future would affect abatement decisions today. For example, the effects of geoengineering could manifest in the form of an increase in free riding on abatement (Barrett, 2008). A second mechanism could be that countries that benefit from higher temperatures may oppose the use of geoengineering. Thus, if the effects of geoengineering are distributed unevenly, it is possible that conflict may arise among nations (Schelling, 1996). With this paper, I expand this literature in two distinct ways. First, by combining the technological advantages of geoengineering with the drawbacks attached to the increase in the free riding effect, I can study the net effects of geoengineering on abatement and wellbeing. Second, by allowing the sequential choice of abatement and geoengineering, I expand the original results that lead to the possibility of conflict, and I propose ways in which the same forces, acting in opposite directions, create more consensus, and possibly less conflict among countries.

The rest of the paper proceeds as follows. Section 2 defines and explains the relation between abatement, geoengineering, and temperature. Section 3 sets out the general model and results. Section 4 defines a linear model in which the best response functions for the two stages of the game are linear. In Section 5, I consider two examples. First, I present the case of identical countries and illustrate the different implications of the free riding effect, the technical substitution effect, and the strategic effect. Second, I examine the case of asymmetric countries and the importance of the strategic effect. Section 6 contains the implications and conclusions of this work.

3.2 Abatement, Geoengineering and Temperature

Climate change policy focuses on the relation between GHG concentrations and temperature changes. Due to the direct link between these two variables, policy has been designed to reach a given level of GHG concentration in order to keep the global average temperature stable at its current levels.

Recently, scientists have proposed ways to alter the climate and artificially achieve a given temperature level by reducing the amount of radiation that reaches the surface of the Earth, independent of the concentration of GHG. These technologies are known as *shortwave geoengineering* (or geoengineering for short) and are meant to alter the increase the reflectiveness of the Earth's atmosphere by injecting reflective particles into the stratosphere.[3]

[3]There are many other possible technologies that can achieve the same outcome (e.g. increasing

Geoengineering technologies are meant to reduce *incoming short wave (or solar) radiation*, which is the radiation reaching the Earth from the Sun. On the other hand, abatement reduces the concentration of GHG which, in turn, increases *outgoing long wave (or terrestrial) radiation*, the radiation leaving the atmosphere (Lenton and Vaughan, 2009). *Radiative forcing* describes how the balance between incoming short wave radiation and outgoing long wave radiation is affected by human activity (IPCC, 2007).[4] Defining the effects of abatement and geoengineering in terms of radiative forcing (R) is useful because the change in mean global temperature (ΔT) is approximately proportional to radiative forcing (IPCC, 2007):

$$\Delta T = \lambda R.$$

where λ is a constant known as the *climate sensitivity parameter*. Formally, if the effects of abatement and geoengineering are measured in terms of their radiative forcing potential, the effective radiative forcing after intervention is given by

$$R = R_0 - A - G.$$

where A and G represent abatement and geoengineering. R_0 is exogenous to the model and it captures the radiative forcing equivalent to a business as usual GHG-concentration. That is, R_0 is the radiative forcing in the absence of abatement and geoengineering. The change in temperature is given by:

$$\Delta T = \lambda(R_0 - A - G). \tag{3.1}$$

the reflectivity of the clouds); however, this technology seems to be the most appropriate from a physical and cost effective point of view (MacCracken, 2006)

[4]The annual global mean flux of solar radiation at the top of the atmosphere is 342 *watts-per-meter-squared*. Of that radiation, approximately 30% is reflected either by the atmosphere or by the surface of the Earth. The remaining 70% of the radiation is absorbed.

Thus, mean global temperature is the outcome of the balance between two types of particles in the atmosphere, GHG and reflective particles (Schelling, 1996).

3.3 The Model

Consider a two-country, two-period partial equilibrium model. The two countries are indexed by $i = \{1, 2\}$. The objective of each country is to minimize its own costs of managing climate change. The objective function of country 1 is given by:[5]

$$\Omega^1 = C^1(a_1) + M^1(g_1) + D^1(A, G) \tag{3.2}$$

where a_1 and g_1 represent country 1's abatement and geoengineering choices, $C^1(.)$ represents the costs of abatement, $M^1(.)$ represents the costs of geoengineering, and $D^1(., .)$ represents the damages associated with climate change, which are a function of the total level of abatement, $A = a_1 + a_2$, and the total level of geoengineering, $G = g_1 + g_2$. The costs of abatement and geoengineering are increasing and convex,[6], $C^1_a > 0$, $C^1_{aa} > 0$, $M^1_g > 0$, and $M^1_{gg} > 0$. Abatement and geoengineering reduce damages from climate change, but they do so at a decreasing rate, $D^1_A < 0$, $D^1_{AA} > 0$, $D^1_G < 0$, and $D^1_{GG} > 0$. Further, I assume $D^1_{AG} > 0$, which implies that the marginal productivity of abatement decreases with geoengineering, and viceversa.

Given the long time delay between the implementation of abatement and any significant reduction in temperatures, abatement can be thought of as a decision variable that affects future outcomes. Geoengineering directly alters the amount of incoming short wave radiation; therefore its effects are immediate. Thus, it is natural

[5]Throughout the paper I calculate the results for country 1; results for country 2 are obtained in a similar way.

[6]I denote partial derivatives as $X_y \equiv \partial X(y, z)/\partial y$ and $X_{yy} \equiv \partial^2 X(y, z)/\partial y^2$.

to model strategic climate intervention in the presence of geoengineering as a two-stage sequential game. In the first stage, countries choose abatement and in the second stage, countries choose geoengineering. While the major results of the model stand when abatement and geoengineering are chosen simultaneously, this timing of events allows me to capture the specific inter-temporal trade-off associated to the possibility of geoengineering becoming available in the future.

3.3.1 Geoengineering Stage

In the second stage, country 1 chooses the level of geoengineering that minimizes its own costs, while holding the decisions of country 2 constant. In this stage, abatement decisions have already been made. The first order condition with respect to geoengineering is

$$M_g^1\left(g_1\right) + D_G^1(A, G) = 0 \tag{3.3}$$

The condition in equation (3.3) is the best response function for country 1 in implicit form. The slope of this best response function is given by:

$$\frac{dg_1}{dg_2} = -\phi_1, \text{ where } \phi_1 \equiv \frac{D_{GG}^1}{M_{gg}^1 + D_{GG}^1} > 0 \tag{3.4}$$

Equation (3.4) implies that an increase in country 2's level of geoengineering causes a reduction in country 1's level of geoengineering. Because geoengineering is a global public good, just as with abatement, geoengineering is under-provided in equilibrium due to free riding. The higher the slope of the marginal costs of geoengineering in country 1, the smaller is the magnitude of the slope of the reaction function for country 1; this provides a larger incentive to free ride on geoengineering in the second stage. If the slope of the marginal damages from geoengineering increase, the free riding effect on geoengineering in the second stage is also increased.

The equilibrium levels of geoengineering in both countries are the implicit solution to equation (3.3), and its counterpart for country 2. This solution can be written as function of A only:

$$g_1 = y^1(A) \text{ and } g_2 = y^2(A) \tag{3.5}$$

Thus, abatement decisions in the first stage affect the choice of geoengineering in the second stage. This is what previous literature refers to when suggesting that free riding on abatement may increase with geoengineering (Barrett, 2008). However, as it will soon become clear, this is but one possibility.

In order to analyze the response in country 1 to an increase in the total level of abatement, totally differentiate (3.3) to obtain:

$$\frac{dg_1}{dA} = -\phi_1 \frac{dg_2}{dA} - \psi_1 \tag{3.6}$$

where $\psi_1 = D^1_{AG}/(M^1_{gg}+D^1_{GG})$. The first term on the right hand side of equation (3.6) represents the movement along the best response function of country 1 induced by the shift of country 2's best response function. The second term represents the shift in country 1's best response function directly induced by the change in abatement. Hence, it is unclear whether or not dg_1/dA is positive or negative. In particular, if the effect of the movement along the best response function is smaller than the effect of the shift, an increase in the first stage level of abatement will result on an decrease in the level of geoengineering in country 1.

The net change in the equilibrium level of geoengineering depends on the response of both countries. To find the net effects of a change in abatement on the equilibrium level of geoengineering in country 1, total differentiate equation (3.3) with respect to g_1, g_2, a_1 and a_2 to obtain a simultaneous equation system that can be solved using

Cramer's rule (see Appendix B). The solution is given by:

$$y_A^1 = \frac{dg_1}{dA} = \frac{\phi_1\psi_2 - \psi_1}{1 - \phi_1\phi_2} \tag{3.7}$$

where $1 - \phi_1\phi_2 > 0$.[7] In particular, whether a change in abatement causes geoengineering to increase or decrease depends on how large the movement along country 1's best response function is, relative to the shift in country 1's best response. Proposition 1 summarizes the general conclusion.

Proposition 1: *The level of geoengineering in country* 1 *increases with a decrease in the total level of abatement implemented in the first stage if and only if the effect of the movement along country 1's best response function, $\phi_1\psi_2$, is lower than the effect of the shift in country 1 best response function, ψ_1*

Proof: Follows directly from (3.7)

When abatement is reduced, the marginal productivity of geoengineering is increased in the second stage for both countries. Proposition 1 shows that the strategic relation between abatement and geoengineering is ambiguous. If the marginal productivity in country 1 increases more than in country 2, then the condition in Proposition 1 is more likely to occur. If the marginal damages in the two countries are very different, it is possible to have one country's equilibrium level of geoengineering increasing when the level of abatement is increased, while the other country's equilibrium level of geoengineering decreases.

[7] To see this replace the definitions for ϕ_1 and ϕ_2 to find $1 - \phi_1\phi_2 = 1 - (D^1_{GG}/(M^1_{gg} + D^1_{GG}))(D^2_{GG}/(M^2_{gg} + D^2_{GG})) = ((M^1_{gg} + D^1_{GG})(M^2_{gg} + D^2_{GG}) - D^1_{GG}D^2_{GG})/((M^1_{gg} + D^1_{GG})(M^2_{gg} + D^2_{GG})) = (M^1_{gg}M^2_{gg} + D^1_{GG}M^2_{gg} + D^2_{GG}M^1_{gg})/((M^1_{gg} + D^1_{GG})(M^2_{gg} + D^2_{GG})) > 0$

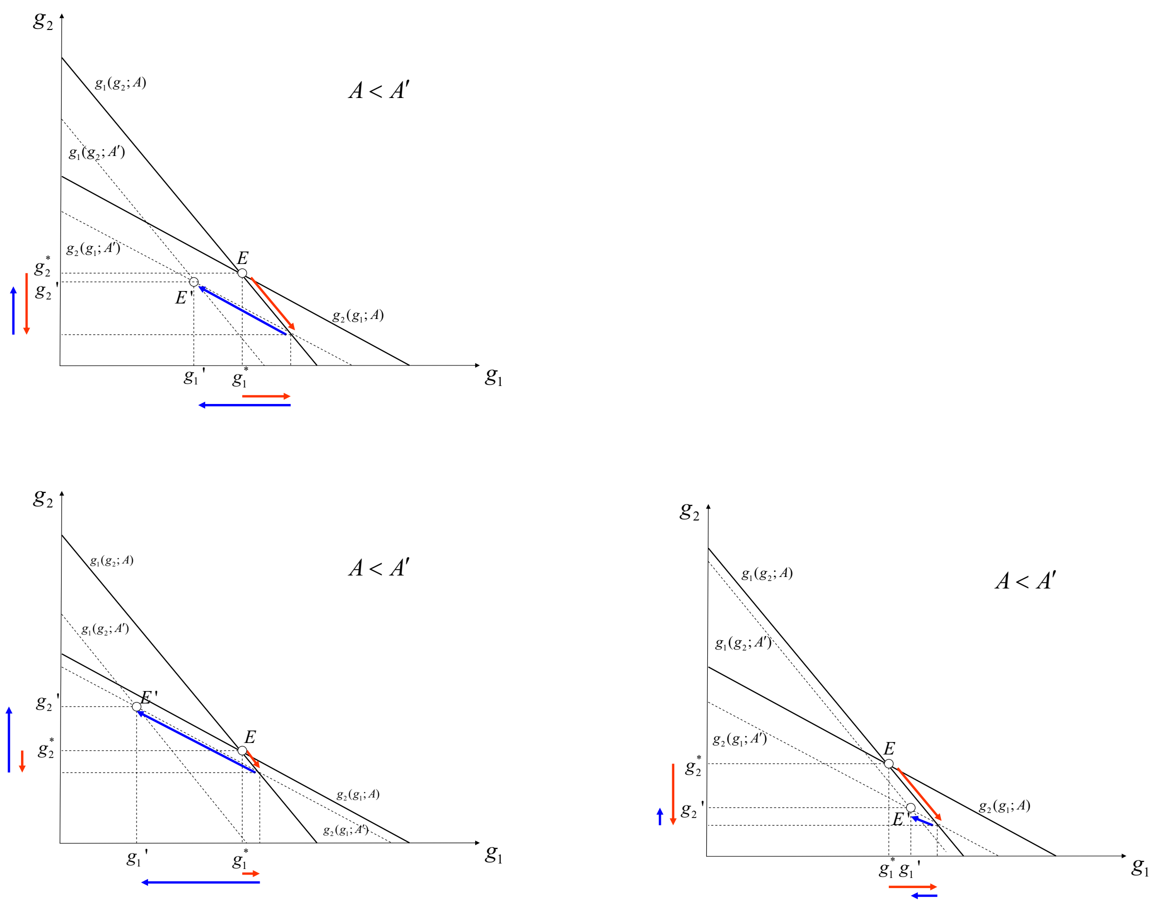

Figure 1. Geoengineering stage best response functions

In Figure 1, I use linear response functions for ease of explanation. Point E denotes the original equilibrium when abatement is equal to A, and point E' is the equilibrium when abatement is equal to $A' > A$. The top panel shows the situation where an increase in abatement leads to a reduction in the equilibrium level of geoengineering in both countries. The bottom-left panel shows the results when the equilibrium level of geoengineering decreases in country 1 but increases in country 2, that is the shift effect is larger in country 2 that is is in country 1, relative to the movement along their best response function. The bottom-right panel shows the reverse situation, the equilibrium level of geoengineering increases in country 1 but decreases in country 2. In the following subsection I analyze the implications of this ambiguity on the first stage choice of abatement.

3.3.2 Abatement Stage

In the first stage, country 1 chooses the level of abatement that minimizes its own costs of managing climate change, taking the choice of the country 2 as given. However, when making their decisions, countries take into consideration the effects of their choice on the level of geoengineering in the second stage. Hence, considering the second stage solution, given by (3.5), the first stage problem becomes

$$\Omega^1 = C^1(a_1) + M^1(y^1(A)) + D^1(A, Y(A)) \tag{3.8}$$

where $Y(A) = y^1(A) + y^2(A)$ and $A = a_1 + a_2$. Using the envelope theorem and equation (3.3), the first order condition for the choice of abatement in country 1 is given by

$$C_a^1(a_1) + D_A^1(A, Y) + D_G^1(A, Y)y_A^2 = 0 \tag{3.9}$$

Before going further with this analysis I restate the main question I set to answer at the beginning of the paper. Does the possibility of geoengineering increase the free riding effect on abatement? To answer this question I study the best response function of country 1 in the first stage.

The slope of the best response function in the first stage is given by:

$$\frac{da_1}{da_2} = -\frac{\nu_1 + \mu_1(y_A^1 + y_A^2) + \mu_1(1 + (\phi_1/\psi_1)(y_A^1 + y_A^2))y_A^2 + (D_G^1/(C_{aa}^1 + D_{AA}^1))y_{AA}^2}{1 + \mu_1(y_A^1 + y_A^2) + \mu_1(1 + (\phi_1/\psi_1)(y_A^1 + y_A^2))y_A^2 + (D_G^1/(C_{aa}^1 + D_{AA}^1))y_{AA}^2} \tag{3.10}$$

where $\nu_1 = D^1_{AA}/(C^1_{aa}+D^1_{AA})$ is the *free riding effect in the absence of geoengineering.*[8] In this specific context, country 1 does not capture the benefits of its abatement in country 2. Thus, the free riding effect on abatement leads to under provision in equilibrium. The term $\mu_1 = D^1_{AG}/(C^1_{aa} + D^1_{AA})$ is the *pure technical substitution effect.* It captures the effects of the shift in the best response function induced by an exogenous change on geoengineering. This effect is introduced by the availability of geoengineering and it consists on the efficient substitution away from abatement and towards geoengineering that any country, in isolation, would make in order to minimize its own costs of managing climate change. The cost minimizing set up suggests a reduction in the level of abatement is efficient up to the point in which the marginal costs of abatement and geoengineering are equalized.

Taking into consideration the strategic interaction across countries, and assuming the effects of the higher orders terms are small, equation (3.10) can be written as:

$$\frac{da_1}{da_2} = -\frac{\nu_1^C + \mu_1^C + \zeta_1}{1+\zeta_1} \tag{3.11}$$

where:

$$\nu_1^C = \frac{\nu_1}{1+\mu_1[y_A^1 + y_A^2]}, \tag{3.12}$$

$$\mu_1^C = \frac{\mu_1[y_A^1 + y_A^2]}{1+\mu_1[y_A^1 + y_A^2]}, \text{ and,} \tag{3.13}$$

$$\zeta_1 = \frac{\mu_1(1+\phi_1/\psi_1[y_A^1 + y_A^2])y_A^2}{1+\mu_1[y_A^1 + y_A^2]} = -\frac{\mu_1\frac{(1-\phi_1)}{\psi_1}y_A^1 y_A^2}{1+\mu_1[y_A^1 + y_A^2]} \tag{3.14}$$

There are three effects that can be identified in equation (3.11). The *free riding*

[8] Assume country 1 makes its abatement decisions without considering the presence of geoengineering. Thus, equation (3.9) can be written as:$C^1_a(a_1) + D^1_A(A, Y) = 0$ Total differentiation with respect to a_1, a_2, and Y, yields: $\frac{da_1}{dY} = -\nu_1\frac{da_2}{dY} - \mu_1$ where $\nu_1 = D^1_{AA}/(C^1_{aa} + D^1_{AA})$ is the free riding effect in the absence of geoengineering and $\nu_1\frac{da_2}{dY}$ captures the movement along country 1's best response function. The term $\mu_1 = D^1_{AG}/(C^1_{aa} + D^1_{AA})$ is the shift of the best response function.

effect, which I denote by ν_1^C, the *technical substitution effect*, which I denote by μ_1^C, and the *strategic effect*, which I denote by ζ_1.[9]

The free riding effect, given by equation (3.12), captures the change in the slope of the best response function in the absence of geoengineering, ν_1. If $(y_A^1 + y_A^2) < 0$, then ν_1^C may be larger than ν_1; representing an *increase* in the free riding effect on abatement. This effect leads to under-provision of abatement, which represents an increase in the costs and temperature, relative to the optimal solution. This increase in the free riding effect will become more pronounced if $(y_A^1 + y_A^2)$ is large in absolute value—that is, if the slope of the marginal cost of geoengineering, M_{GG}^1, increases or if the slope of the marginal damages increases, D_{GG}^1, decreases. When geoengineering is available, country 1 reduces its level of abatement because country 2 has two instruments to manage climate change; country 2 has more capacity to absorb country 1's under-provision of abatement.

The technical substitution effect, given by equation (3.13), captures the technical substitution from abatement towards geoengineering due to the endogenous choice of geoengineering. If $(y_A^1 + y_A^2) < 0$ this effect is negative, which reduces the slope of the best response function in the first stage. This represents a *decrease* in the free riding effect on abatement. As before, the magnitude of this effect depends on the magnitude and the sign of $(y_A^1 + y_A^2)$. This effect is in essence efficient. When geoengineering becomes available, it is optimal for country 1 to reduce its level of abatement. However, the strength of this technical substitution effect is limited by the fact that any substitution away from abatement has implications on the decisions

[9]The magnitude of the slope is less than one because $\nu_1 < 1$, which makes $\nu_1^C + \mu_1^C < 1$. The denominator is given by $1+\zeta_1$ which I assume to be positive. This is the Routh-Hurwicz condition for best response function stability of the abatement game, and it is standard to make this assumption in these type of games (Brander and Spencer, 1983; Dixit, 1986)

of country 2.

The strategic effect, given by equation (3.14), could increase the free riding effect in abatement depending on the signs of y_A^1 and y_A^2. In particular, if y_A^1 and y_A^2 have the same sign, the strategic effect tends to reduce the free riding effect on abatement. However, the strategic effect also affects the denominator in equation (3.11). If y_A^1 and y_A^2 have the same sign, the denominator is less than one and the net effect is an increase in the slope of the first stage reaction function. The opposite is true when y_A^1 and y_A^2 have opposite signs. The direction of change in the free riding effect induced by the strategic effects is also ambiguous. In particular, if geoengineering is a substitute for abatement, country 1 has an incentive to reduce its level of abatement and induce country 2 to increase its level of geoengineering. However, if geoengineering is consider to be a public bad, it will create an incentive to increase its level of abatement in order to deter the use of geoengineering in the second stage. In this case, abatement and geoengineering become strategic complements. This situation can lead to an equilibrium in which there is too much abatement relative to optimal solution.

Notice that the ambiguity in the results on the free riding on abatement arises because of the impossibility to sign the derivative of geoengineering with respect to abatement: it is not clear whether abatement and geoengineering are strategic substitutes or strategic complements.

To summarize, geoengineering introduces three different effects: it modifies the free riding effect, it introduces the technical substitution effect, and it introduces the strategic effect. These last two effects may work in the opposite direction of an increase in the free riding effect, depending on whether y_A^1 and y_A^2 are positive or negative. Thus, the net effect of geoengineering in the choice of abatement is undetermined. In the sections below I analyze examples in which the net effects are clearly identifiable.

3.4 The Linear Model

In the previous section I showed that the overall effect geoengineering has on the free riding effect in abatement is ambiguous. Here, I introduce a simpler version of the model where marginal abatement costs, marginal geoengineering costs, and marginal damages are linear functions, which allows me to find closed form solutions to the expressions developed in the previous section, and it also allows me to develop more intuition about the results of the general model.

3.4.1 Costs and Damages

The costs of abatement and geoengineering are assumed to be quadratic and are given by:

$$C^1(a_1) = \frac{1}{2}\alpha_1 a_1^2 \text{ and } M^1(g_1) = \frac{1}{2}\gamma_1 g_1^2 \tag{3.15}$$

where α_1 and γ_1 are positive constants representing the slopes of the marginal cost of abatement and geoengineering, respectively.

There are two sources of economic damages: temperature damages and geoengineering damages. Temperature damages are those caused by the direct change in temperature as defined in equation (3.1), e.g. the sea level rising. The other source of economic damages are caused by the possible side effects from the implementation of geoengineering; e.g. ozone decay and changes in precipitation patterns.[10] I introduce

[10]The geoengineering technology I describe in this paper addresses only temperature related damages, while leaving other damages untreated (i.e. ocean acidification). For simplicity I do not consider

damages from geoengineering to account for the possibility of asymmetric impacts of geoengineering (Barrett, 2008; Schelling, 1996; Victor, 2008; Victor et.al., 2009). The damage function is given by:

$$D^1(A,G) = \frac{1}{2}\delta_1(R_0 - A - G)^2 + \frac{1}{2}\rho_1 G^2 \tag{3.16}$$

where δ_1 and ρ_1 are positive constants. With this definition of damages, the second order derivatives are $D^1_{AA} = \delta_1 > 0$, $D^1_{GG} = \delta_1 + \rho_1 > 0$, and the cross derivative $D^1_{AG} = \delta_1 > 0$. Therefore, in the linear model, ϕ_i, ψ_i, μ_i, and ν_i are all positive constants.

Replacing (3.15) and (3.16) in (3.2), I obtain country 1's objective function for the linear case:

$$\Omega^1 = \frac{1}{2}\alpha_1 a_1^2 + \left[\frac{1}{2}\gamma_1 g_1^2 + \frac{1}{2}\delta_1\left(R_0 - A - G\right)^2 + \frac{1}{2}\rho_1 G^2\right] \tag{3.17}$$

Notice that Ω^1, and the equivalent Ω^2, are increasing and convex in a_1, a_2, g_1 and g_2; thus an interior equilibrium exists and it is stable.

3.4.2 Geoengineering Stage

One of the benefits of the linear model is that equation (3.3) can now be solved explicitly for g_1 as a function of g_2 and A. The second stage best response function is given by:

$$g_1(g_2; A) = -\phi_1 g_2 + \psi_1\left[R_0 - A\right] \tag{3.18}$$

where $\phi_1 = (\delta_1 + \rho_1)/(\gamma_1 + \delta_1 + \rho_1)$ and $\psi_1 = \delta_1/(\gamma_1 + \delta_1 + \rho_1)$. The size of the movement along country 1's best response function induced by a change in abatement is given

damages from climate change different to those caused directly by global warming. For a complete treatment of the different damages in a non-strategic environment please refer to Moreno-Cruz and Smulders (2009) by $\phi_1 \cdot (dg_2/dA)$ and the size of the shift is given by ψ_1. The two terms, ϕ_1 and ψ_1, differ only by the presence of ρ_1 in the numerator of ϕ_1. Thus, for large values of ρ_1, the effect of the movement along the best response curve will tend to dominate the effect of the shift. However, the effects of a change in abatement in equilibrium also depend on the magnitude of the shift in country 2's best response function, $dg_2/dA = \delta_2/(\gamma_2 + \delta_2 + \rho_2)$.

The best response function defined in (3.18) and its equivalent for country 2, can be solved for g_1 and g_2 conditional on $A = a_1 + a_2$. Specifically, the level of geoengineering can be written as:

$$y^1(A) = -\left[\frac{\phi_1\psi_2 - \psi_1}{1 - \phi_1\phi_2}\right](R_0 - A) \tag{3.19}$$

Replacing the definitions of ϕ_1, ϕ_2, ψ_1, and ψ_2, I obtain the following expression for $\partial y^1/\partial A$:

$$y_A^1 = \frac{\delta_2\rho_1 - \delta_1(\gamma_2 + \rho_2)}{\gamma_1\gamma_2 + \gamma_1(\delta_2 + \rho_2) + \gamma_2(\delta_1 + \rho_1)} \tag{3.20}$$

An increase in the abatement level of either country can increase or decrease the level of geoengineering in the second stage, depending on the sign of (3.20).

Proposition 1': *The second stage level of geoengineering in country* 1 *increases when the total level of abatement decreases in the first stage if and only if the movement along country 1's best response function is small relative to the shift of its best response.*

Proof: Follows directly from (3.20).

If the condition in Proposition 1' holds, then an increase in the total level of abatement decreases the equilibrium level of geoengineering. Simply stated, Proposition 1' says that, if the slope of the marginal damages of temperature in country 1 is small, relative to the marginal damages of temperature in country 2, country 1 has an incentive to reduce its level of abatement to force an increase in the level of geoengineering in the second stage. The extent to which country 1 reduces its level of abatement depends on how large are its marginal damages of geoengineering. If the marginal damages of geoengineering are low, relative to those of country 2, then the result is a higher level of geoengineering in the second stage.

3.4.3 Abatement Stage

The slope of the best response function in the first stage is given by (the same as in equation (3.11)):

$$\frac{da_1}{da_2} = -\frac{\nu_1^C + \mu_1^C + \zeta_1}{1 + \zeta_1} \tag{3.21}$$

The free riding effect, the technical substitution effect, and the strategic effect can be calculated explicitly for the linear case and they are given by:

$$\nu_1^C = \frac{\delta_1(\gamma_1\gamma_2 + \gamma_1(\delta_2 + \rho_2) + \gamma_2(\delta_1 + \rho_1))}{\alpha_1(\gamma_1\gamma_2 + \gamma_1(\delta_2 + \gamma_2) + \gamma_2(\delta_1 + \gamma_1)) + \delta_1(\gamma_1\gamma_2 + \gamma_2\rho_1 + \gamma_1\rho_2)}, \tag{3.22}$$

$$\mu_1^C = -\frac{\delta_1(\delta_1\gamma_2 + \delta_2\gamma_1)}{\alpha_1(\gamma_1\gamma_2 + \gamma_1(\delta_2 + \gamma_2) + \gamma_2(\delta_1 + \gamma_1)) + \delta_1(\gamma_1\gamma_2 + \gamma_2\rho_1 + \gamma_1\rho_2)}, \text{ and} \tag{3.23}$$

$$\zeta_1 = \frac{\delta_1((\delta_1\gamma_2 + \delta_1\rho_2 - \delta_2\rho_1)(\delta_2\gamma_1 + \delta_2\rho_1 - \delta_1\rho_2))/(\gamma_1\gamma_2 + \gamma_1(\delta_2 + \rho_2) + \gamma_2(\delta_1 + \rho_1))}{\alpha_1(\gamma_1\gamma_2 + \gamma_1(\delta_2 + \gamma_2) + \gamma_2(\delta_1 + \gamma_1)) + \delta_1(\gamma_1\gamma_2 + \gamma_2\rho_1 + \gamma_1\rho_2)}. \tag{3.24}$$

Whether the combined effects result in an increase or a decrease of the free riding effect is not clear; it depends on the magnitudes of the different effects, which in turn relies on the parameters of the model.

Each one of the effects is affected differently by the underlying parameters of the model. In particular, an increase in the marginal damages from temperature in either country, causes an increase in ν_1^C. It also increases the productivity of abatement relative to that of geoengineering, therefore decreasing the technical substitution effect,

μ_1^C. Similarly, when the marginal damages from geoengineering increase for either country, there is a reduction in μ_1^C, and also a reduction in the free riding effect, ν_1^C. Finally, whether ν_1^C or μ_1^C increase or decrease with changes in the marginal costs of geoengineering depends on the sign of y_A^1. Specifically, an increase in the marginal costs of geoengineering in country 1, γ_1, increases ν_1^C and reduces μ_1^C only when $y_A^1 < 0$, because the ability of country 1 to substitute abatement towards geoengineering is reduced. An increase in the marginal costs of abatement in country 1, α_1, increases the technical substitution effect, and reduces the free riding effect; however the net effects on the equilibrium level of abatement remains unclear.

The effects of changes in the parameters on the strategic effect are ambiguous because these changes move y_A^1 and y_A^2 in opposite directions. Thus, the net change depends on the relative magnitudes of y_A^1 and y_A^2. For example, an increase in the costs of geoengineering in country 2 causes an increase in y_A^1 and a decrease in y_A^2, if $y_A^2 > 0$. The opposite is true if $y_A^2 < 0$. In the two examples I develop below, the relative magnitude of the changes in y_A^1 and y_A^2 are fully determined, and the ambiguity of the strategic effect, ζ_1, is eliminated.

3.5 Two examples

In order to study the importance of the different effects I will proceed with two examples. First, I analyze the case of identical countries. In this case, $y_A^1 = y_A^2 = y_A < 0$. Moreover, the shifts of the best response functions are identical across countries. Second, I analyze the case of asymmetric countries. In particular, I assume that only country 1 is able to implement abatement in the first stage and only country 2 is able to implement geoengineering in the second stage. In this case, $y_A^1 = 0$ and $y_A^2 < 0$, thus the ambiguity on the effects induced by the free riding, technical substitution, and strategic effects is eliminated.

3.5.1 Identical Countries

Assume that countries are identical. In this case, equation (3.20) is reduced to:

$$y_A = -\frac{\delta}{\gamma + 2(\delta + \rho)} \tag{3.25}$$

which is always negative. When countries are identical, the movement along and the shift of the best response functions are the same for both countries. Thus, the strategic effects are equalized and I can concentrate on the importance of the technical substitution effect, relative to the strategic substitution effect. I illustrate these results in Figure 2.

Point E in Figure 2 is the equilibrium level of geoengineering in the second stage as a function of abatement, for given A. The dotted lines represent the case where there is an increase in the level of abatement. An increase in the level of abatement by either country decreases marginal damages in the second stage, shifting both reaction functions inward, and in equal proportions. By doing so, it decreases the marginal productivity geoengineering in both countries, resulting in a lower equilibrium level of geoengineering. The new level of geoengineering evaluated at $A' > A$ is marked by point E'.

The equilibrium level of abatement in either country is given by:

$$a^* = \frac{\nu^C + \mu^C + \zeta}{1 + \nu^C + \mu^C + 2\zeta} R_0 \tag{3.26}$$

where $\nu^C = \delta(\gamma + 2(\delta + \rho))/(\alpha(\gamma + 2(\delta + \rho)) + \delta(\gamma + 2\rho))$, $\mu^C = 2\delta^2/(\alpha(\gamma + 2(\delta + \rho)) + \delta(\gamma + 2\rho))$, and $\zeta = -(\gamma\delta^2/(\gamma + 2(\delta + \rho)))/(\alpha(\gamma + 2(\delta + \rho)) + \delta(\gamma + 2\rho))$ are the free riding, the technical substitution, and the strategic effects when countries are identical.

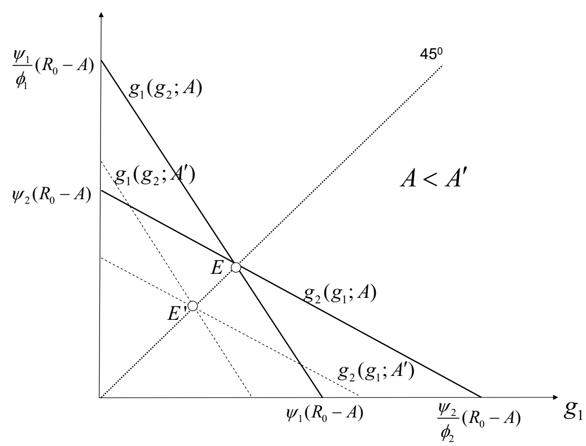

Figure 2. Geoengineering stage best response functions

Using the result in equation (3.26), it is easier to analyze the net effects of introducing geoengineering in the economy. I compare results of the equilibrium level of abatement to the situation in which geoengineering is not available. Define the equilibrium level of abatement in the absence of geoengineering as a^{**}, which is given by:

$$a^{**} = \frac{\nu}{1+\nu} R_0, \text{ where } \nu = \frac{\delta}{\alpha + \delta} \tag{3.27}$$

Proposition 2: *If countries are identical, there is a reduction in the level of abatement relative to the case in which geoengineering is not available in the economy,* $a^{*} < a^{**}$.

Proof: see Appendix B

Proposition 2 shows that when countries are identical, the level of abatement in the first stage is reduced. This is the reason why it has been suggested in the literature that making geoengineering available is a problem. However, this reduction is the composition of two different shifts of the best response function. The free riding effect and the strategic effect will certainly represent an increase in the costs of society; but the the technical substitution effect works in the opposite direction.

In Figure 3, the dashed lines represent the best response functions when there is no geoengineering, the dot-dashed lines represent the shift of the best response functions due to the free riding and technical substitution effects, and the solid lines represent the best response functions including the strategic effects. The reduction from a^{**} to a^{***} is due to the combined effects of the free riding and technical substitution effects. In this case, both effects combine to allow for a reduction in the level of abatement, relative to the case of no geoengineering, $a^{**} > a^{***}$. The difference between a^{***} and a^{*} is due to the strategic effect which further reduces the level of abatement.

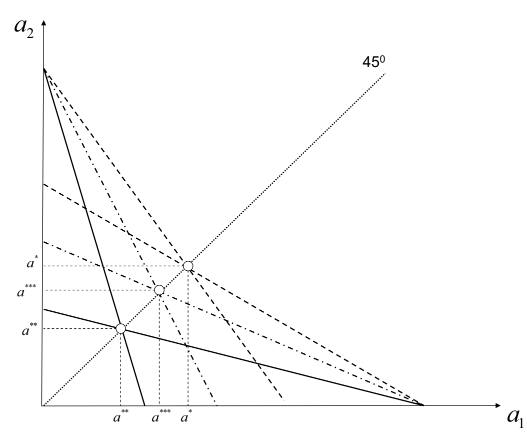

Figure 3. Direct effect and strategic effect with identical countries.

Although the level of abatement is reduced, the technical substitution effect on wellbeing may dominate the free riding and strategic effects, thus improving wellbeing.

In this analysis going forward I assume an increase in wellbeing is equivalent to a reduction in temperature and to a reduction in the total costs of managing climate change.

The effects on temperature are ambiguous. When abatement is reduced geoengineering increases in both countries, as it is shown in Figure 2. Thus, the net change in temperature is determined by the strength of the strategic effect relative to the technical substitution effect. In particular, define T^* as the temperature level in the sequential equilibrium and T^{**} the temperature level when geoengineering is not available.

Proposition 3: *If the marginal abatement costs are small relative to the costs of increasing geoengineering in the second stage, $\alpha < -\gamma y_A$, then the equilibrium temperature with geoengineering is higher than without geoengineering, $T^* > T^{**}$.*

Proof: see Appendix B

Proposition 3 shows that the incentive to reduce abatement created by the availability of geoengineering in the second stage may be so large as to cause an increase in the surface temperature of the planet. In particular, the condition $-\gamma y_A$ increases when γ increases; thus, for large values of marginal costs of geoengineering, γ, temperatures in the equilibrium with geoengineering may be higher than in the equilibrium without geoengineering. The opposite is true if, for example, the marginal damages from temperature, δ, are very large.

Introducing geoengineering can be optimal as long as the total costs of managing climate change are reduced, independent of whether or not temperature increases. Recall that the strategic effect is a function of the technical substitution effect, previously defined as μ_1^C. Therefore, the reduction in abatement due to the technical substitution effect could reduce total costs — that is, the costs of implementing the equilibrium policy and the costs associated with the damages from temperature and geoengineering will be lower when geoengineering is available in the economy.

Proposition 4: *The total costs of climate change are always lower with geoengineering than without geoengineering.*

Proof: see Appendix B

When countries are identical, if damages increase, they must do so less than proportionally to the reduction in the costs of abatement. This occurs because the effects are equalized across countries, which makes the technical substitution effect more important relative to the strategic effect. Thus, the reduction in the cost of implementing the equilibrium strategy are greater than the possible increases in damages created when temperature rises.

Summarizing, when countries are identical, the introduction of geoengineering reduces costs but may increase temperatures. The net effects on wellbeing are not clear. However, from a cost minimizing point of view, geoengineering should be available. As with any other global public good, the asymmetries across countries command the extend to which the introduction of geoengineering is beneficial. In the next section I show that the type of asymmetry affects the implementation of geoengineering in such a way that its availability can increase or decrease the level of abatement in equilibrium.

3.5.2 Asymmetric Countries

Expanding the analysis to the case of asymmetric countries is not trivial. The net change in abatement, temperature or total costs depends on the relative magnitudes of the different effects described previously. When countries are symmetric the slope of the second stage reaction function is always negative, and the net effects are equalized across countries. That is, the shift along the best response functions in both countries is the same. As a result, the equilibrium level of abatement is lower as are overall costs.

In this section, I eliminate the technical substitution effect and concentrate on the strategic effect. To do so, I assume only country 1 can afford to do abatement and only country 2 can perform geoengineering.[11] Clearly, in the real world most countries could do both abatement and geoengineering. Hence, this case is best thought of as a limiting case where the costs of abatement for country 2 are too large *relative* to the costs of geoengineering (e.g. China or India), while the costs of geoengineering are too large *relative* to the costs of abatement for country 2 (e.g. the European Union where geoengineering could be thought of as political burden).

By construction, no individual country can do both abatement and geoengineering; therefore, all substitution from abatement towards geoengineering is solely due to the strategic interaction across countries.[12] These assumptions imply that $\phi_2 = 0$ in equation (3.18):

$$g_2 = \psi_2(R_0 - a_1) \tag{3.28}$$

which is the equilibrium level of geoengineering in country 2 as a function of the level of abatement in country 1. In this case, $\psi_2 = \delta_2/(\gamma_2 + \delta_2 + \rho_2)$. The slope of country

[11]The idea here is that the country that cannot implement abatement has a greater incentive to research geoengineering technologies while the other country has no incentive to do so. One could also think that the country that can implement abatement has a constituency that rejects the implementation of geoengineering. These are possibilities that are in line with this story; however, this is just a theoretical construct to show the opposite extreme of the spectrum compared to the case of perfect symmetry.

[12]This is equivalent to setting $\gamma_1 + \rho_1$ prohibitively high, and α_2 prohibitively high as well.

2's best response function increases with the marginal damages from temperature in country 2, δ_2, and decreases with the marginal costs of geoengineering, γ_2, or the marginal damages from geoengineering, ρ_2.

When country 1 makes its decision on how much abatement to implement, it takes into consideration the relation established in (3.28). Thus, the best response function for country 1 is given by:

$$a_1 = \mu_1^C R_0 - \zeta_1 g_2 \tag{3.29}$$

where $\mu_1^C = \mu_1(1 + g_{a_1}^2)/(1 + \mu_1 g_{a_1}^2)$ and $\zeta_1 = \mu_1(1 + (1 + (\rho_1/\delta_1))g_{a_1}^2)/(1 + \mu_1 g_{a_1}^2)$. As before, $\mu_1 = \delta_1/(\alpha_1 + \delta_1)$. For the case of asymmetric countries, $\mu_1^C > 0$ as $1 + g_{a_1}^2 = (\gamma_2 + \rho_2)/(\gamma_2 + \delta_2 + \rho_2) > 0$. However, ζ_1 can be either positive or negative, depending on the size of the damages from temperature and the damages from geoengineering in country 1. This result is captured in the next proposition.

Proposition 1": *Geoengineering decreases when abatement is increased if and only if the marginal costs of substituting one unit of abatement for one extra unit of geoengineering is larger in country 1 than it is in country 2.*

Proof: $\zeta_1 = [\delta_1 + (\delta_1 + \rho_1)g_{a_1}^2]/(\alpha_1 + \delta_1 + \delta_1 g_{a_1}^2)$. From equation (3.28): $g_{a_1}^2 = -\delta_2/(\gamma_2 + \delta_2 + \rho_2)$. Thus, $\zeta_1 > 0$ when $\delta_2/(\gamma_2 + \rho_2) < \delta_1/\rho_1$ and $\partial a_1/\partial g_2 < 0$.

In this set-up, the ambiguity in the results is introduced by the sequentiality of the problem. If decisions by country 1 and country 2 were made simultaneously, we should always have $\partial a_1/\partial g_2 < 0$.

If $\zeta_1 > 0$, then an increase in the level of abatement causes a decrease in the level of geoengineering. I show this result in Figure 4. In this case, it is optimal for country 1 to reduce its level of abatement to induce an increase in geoengineering. Thus, the costs of managing climate change in country 2 will increase.

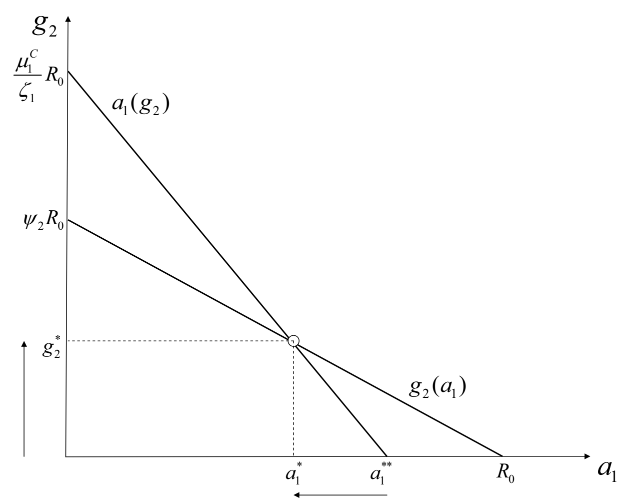

Figure 4. Asymmetric case. Abatement and geoengineering are substitutes.

Next, I compare the equilibrium level of abatement in the presence of geoengineering to the case where there is no geoengineering. I find conditions where geoengineering induces higher equilibrium levels of abatement.

When countries are asymmetric, the equilibrium level of abatement in country 1 is given by:

$$a_1^* = \frac{\mu_1^C - \zeta_1\psi_2}{1 - \zeta_1\psi_2}R_0 \tag{3.30}$$

The level of abatement implemented by country 1 in the absence of geoengineering is given by:

$$a_1^{**} = \nu_1 R_0 \tag{3.31}$$

Assume that $\delta_2\rho_1 > \delta_1(\gamma_2 + \rho_2)$; this implies abatement increases with geoengineering. In the next proposition I show the condition that needs to hold in order to have more abatement in the equilibrium with geoengineering, compared to the equilibrium without geoengineering.

Proposition 5: *Abatement with geoengineering is larger than without geoengineering, $a_1^* > a_1^{**}$ if and only if the marginal damages from temperature in country 1 are lower relative to country 2, $\delta_1 < \delta_2$, and the marginal damages from geoengineering in country 1 are larger than in country 1, $\rho_1 > \rho_2/(2(\gamma_2 + \rho_2) + \delta_2)$*

Proof: see Appendix B

Proposition 5 implies that if the damages from geoengineering in country 1 are high enough, or the damages from temperature are small enough in country 1; then, country 1 has a greater incentive to increase its abatement level in order to reduce the level of geoengineering chosen by country 2. In particular, in the limiting case with $\gamma_2 + \rho_2 = 0$, the condition in Proposition 5 holds when $\delta_1 < \rho_1$.

For even larger asymmetries, the presence of geoengineering could increase abatement levels even beyond first best levels in the absence of geoengineering. The first best level of abatement in the absence of geoengineering is equal to the solution of the following problem:

$$\min_{\{a_1\}} \frac{1}{2}\alpha_1 a_1^2 + \frac{1}{2}(\delta_1 + \delta_2)(R_0 - a_1)^2$$

The solution is given by:

$$a_1^{FB} = \frac{\delta_1 + \delta_2}{\alpha_1 + \delta_1 + \delta_2} R_0 \tag{3.32}$$

Using this definition, Proposition 6 shows that there is a combination of parameters for which the presence of geoengineering induces higher levels of abatement, relative to the first best case.

Proposition 6: *The equilibrium level of abatement with geoengineering is larger than the optimal level of abatement without geoengineering,* $a_1^* > a_1^{FB}$*, if and only if the marginal damages from temperature in country 1 are* ***substantially*** *lower than in country 2,* $\delta_1 << \delta_2$*, and the marginal damages of geoengineering in country 1 are* ***substantially*** *larger than in country 2,* $\rho_1 >> \rho_2$*.*

Proof: see Appendix B

The condition in Proposition 6 implies that the possibility of achieving first best levels of abatement depends radically on the asymmetries of damages. For example, if total costs of geoengineering in country 2 are very low ($\gamma_2 + \rho_2 = 0$), then the condition in Proposition 6 holds when $\delta_1 + \delta_2 < \rho_1$. That is, when marginal damages from geoengineering in country 1 are larger than the sum of the marginal damages from temperature in the two countries. I show this result in Figure 5.

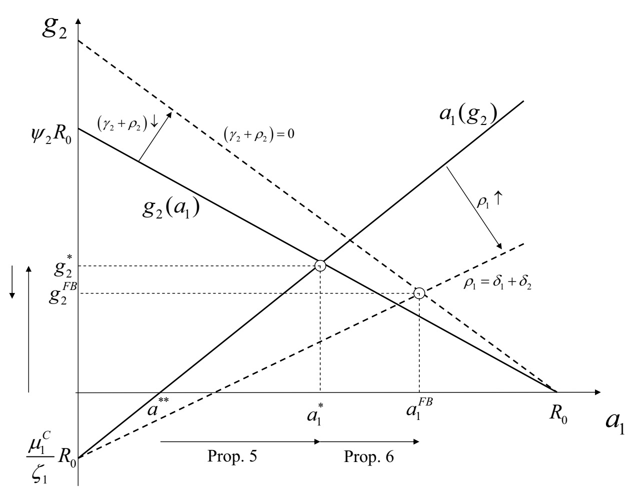

Figure 5. Asymmetric case. Abatement and geoengineering are complements.

The results in Propositions 5 and 6 are more than a theoretical curiosity. Large asymmetries in the damages from climate change have been documented previously in the literature. In particular, the IPCC's Third Assessment Report compares different studies and shows that if surface temperature increases by 2.5^{o}C, countries like Russia will gain 0.7% in their GDP, while regions of the world like India or Africa will suffer damages on the order of 4% to 5% of GDP (IPCC, 2001). Thus, it is possible for the first part of the conditions in Proposition 5 and 6 to hold. The second part of the condition in Propositions 5 and 6 are more difficult to assess. However, recent computer experiments on the effects of geoengineering schemes in the hydrological cycle suggest that the damages from geoengineering will be highly asymmetric (Matthews and Caldeira, 2007; Bala et.al. 2008). Thus, if the damages from precipitation are larger in some countries relative to others, the second part of the condition in Propositions 5 and 6 is more likely to hold.

Following the previous two propositions, it is easy to show that for large asymmetries in the damages from climate change and geoengineering, the cost to country 1 would increase beyond the costs associated with the first best levels of abatement absent geoengineering. This suggest that country 2 could attain a better outcome in a negotiation if it could credibly threaten to engage in geoengineering. Country 1 now has a real incentive to commit to higher levels of abatement or else country 2 would resort to geoengineering strategies in the second stage, making country 1 worse off.

To summarize, when countries are asymmetric two possibilities arise. First, if the asymmetries from climate change and geoengineering are parallel — that is, if the winners with climate change are winners with geoengineering — geoengineering further reduces wellbeing in countries that were losers from climate change. In this case, abatement levels are inefficiently low. However, if the asymmetries from climate change are orthogonal — that is, is the winners with climate change are losers from geoengineering — and if the costs of abatement are not prohibitively high, it is possible to have more abatement when geoengineering is available. In this situation, countries that were originally losers from climate change increase their wellbeing at the expense of a reduction in the wellbeing of other countries.

3.6 Conclusions

This paper has shown that geoengineering does not necessarily increase the well known free riding effect on abatement. In fact, under asymmetry, it is possible that the prospect of geoengineering may induce higher levels of abatement; even inefficiently high levels of abatement relative to the first best in the absence of geoengineering.

The analysis was performed using a model that is purposely simple. I have done so to concentrate on the strategic interaction between countries. Although some of my results are contrary to standard results in the literature, the method I use is very familiar. In fact, these results follow from seriously considering the differences between abatement and geoengineering, which lend themselves to the application of the same standard two-stage equilibrium methods as those used in the analysis of capacity building (or R&D) and output.

This paper suggests that strategic effects play a major role in determining the impact of geoengineering on the analysis of climate change. In particular, the presence of geoengineering may introduce new leverage that favors developing countries in future negotiations on climate change.

At this point in time, it is difficult to conclude whether geoengineering technologies are essential for dealing with climate change; however, the same lack of evidence makes it very difficult to conclude that the best option is to preclude their use. A serious understanding of the interaction between geoengineering and abatement, both theoretically and empirically, is necessary to be able to determine whether or not geoengineering is worth considering as a tool to manage climate change. This paper is a step towards this understanding.

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Appendix A

Appendix Chapter 1

A.1 Proofs

A.1.1 Proof of Lemma 1

From equation (1.1) and (1.2):

$\delta_S(S,G) = \delta^1_S + \delta^1_T t_S > 0;$

$\delta_G(S,G) = \delta^1_T t_G + \delta^2_G = -\lambda\delta^1_T > 0.$

$\delta_{SS}(S,G) = \delta^1_{SS} + \delta^1_{TS} t_S + \delta^1_{TT}[t_S]^2 + \delta^1_T t_{SS} < 0;$

$\delta_{GG}(S,G) = \delta^1_{TT}[t_G]^2 + \delta^1_T t_{GG} + \delta^2_{GG} = \delta^1_{TT}\lambda^2 < 0$; and

$\delta_{SG}(S,G) = \delta^1_{ST} t_G + \delta^1_{TT} t_S t_G + \delta^1_T t_{SG} = -\delta^1_{ST}\lambda - \delta^1_{TT}\beta\lambda^2/S > 0.$

A.1.2 Proof of Lemma 2

From equation (1.1) and (1.4):

$D_{SS}(S,G) = d^1_{SS} + d^2_{TT}[T_S]^2 + d^2_T T_{SS} = d^1_{SS} + [d^2_{TT}\beta\lambda - d^2_T](\beta\lambda/S^2)$. This proves (d).

$D_{GG}(S,G) = d^2_{TT}[T_G]^2 + d^2_T T_{GG} + d^3_{GG} = d^2_{TT}\lambda^2 + d^3_{GG} > 0$; and

$D_{SG}(S,G) = d^2_{TT} T_S T_G + d^2_T T_{SG} = -d^2_{TT}\beta\lambda^2/S < 0$. This proves (c).

For $S \gtrless \bar{S}$ and $T \gtrless \bar{T} \Leftrightarrow S \gtrless S_0 \exp(\bar{T}/\lambda\beta) \equiv S^{\bar{T}}$, we have $D_S(S,0) = d^1_S + d^2_T T_S \gtrless 0$. Since $D_S(S,0)$ is continuous, we have:

$D_S(S,0) = 0$ for S equal to some value $S^{ZDN} \in \left[\min\{\bar{S}, S^{\bar{T}}\}, \max\{\bar{S}, S^{\bar{T}}\}\right]$.

This value is unique if $D_{SS} > 0$. This proves (a).

By construction $D_G(S, \Gamma(S)) = 0$.

First, since for $S = S^{\bar{T}}$, we have $T = \bar{T}$ and $D_G(S,0) = d^2_T(\bar{T})T_G + d^3_G(0) = 0$, it follows that $\Gamma(S^{\bar{T}}) = 0$.

Second, totally differentiating $D_G(S, \Gamma(S)) = 0$, we find $\Gamma'(S) = -D_{GS}/D_{GG}$. From part (c), $-D_{GS}/D_{GG} > 0$.

Third, since $D_{GG} > 0$ and $D_G(S, \Gamma(S)) = 0$, we have $D_G > 0$ for $G > \Gamma(S)$. This proves (b).

A.1.3 Derivation of optimal policy

The Lagrange function of the cost minimization problem is given by the following expression:

$$\pounds = C(A) + M(G) + D(S,G) + \tau\left(\bar{S}^{LF} - A - S - \delta(S,G)\right) \tag{A.1}$$

The first order conditions are given by:

$$\pounds_A = 0 \Leftrightarrow C_A(A) - \tau = 0 \tag{A.2}$$

$$\pounds_G = 0 \Leftrightarrow M_G(G) + D_G(S,G) - \tau\delta_G(S,G) = 0 \tag{A.3}$$

$$\pounds_S = 0 \Leftrightarrow D_S(S,G) - \tau\left(1 + \delta_S(S,G)\right) = 0 \tag{A.4}$$

From condition (A.2) we find (1.7) and from (A.4) we find (1.8). Finally, eliminating τ between (A.3) and (A.4), we find (1.11).

A.1.4 Proof of lemma 3

Total differentiation of (1.11) yields $\frac{dG}{dS} = \frac{\delta_G D_{SS} + \delta_{GS} D_S - (1+\delta_S) D_{GS} - (M_G + D_G)\delta_{SS}}{(1+\delta_S)(M_{GG}+D_{GG}) + (M_G+D_G)\delta_{SG} - \delta_G D_{SG} - \delta_{GG} D_S}$. Invoking Lemma 1 and Lemma 2 is easy to show that $\frac{dG}{dS} > 0$. This proves lemma 3, part (i). If $T(S,0) = \bar{T}$ and $G = 0$, we have $M_G(G) = D_S(S,G) = 0$ so that the net marginal gains from introducing geoengineering are zero (all terms in (1.11) are zero); this proves (ii).

A.1.5 Proof of Proposition 1

Total differentiation of (1.7) and (1.8) gives: $\frac{dS}{dG} = \frac{-C_{AA}\delta_G(1+\delta_S) - (D_{SG} - C_A\delta_{SG})}{D_{SS} - C_A\delta_{SS} + C_{AA}(1+\delta_S)^2} > 0$ if and only if (1.10) holds. $\frac{d\tau}{dG} = \frac{C_{AA}(1+\delta_S)(D_{SG} - C_A\delta_{SG}) - C_{AA}\delta_G(D_{SS} - C_A\delta_{SS})}{D_{SS} - C_A\delta_{SS} + C_{AA}(1+\delta_S)^2} < 0$, $\frac{dA}{dG} = \frac{1}{C_{AA}}\frac{d\tau}{dG} < 0$. Given existence of an interior solution requires the second order condition to be satisfied, which requires that the denominator term, $D_{SS} - C_A\delta_{SS} + C_{AA}(1+\delta_S)^2$, to be positive. QED

A.1.6 Proof of Proposition 2

Since $D_S(S^{ZD}, g(S^{ZD})) = 0$ by construction and $D_{SS}(S, g(S)) > 0$ for all S, we have $D_S(S, g(S)) > 0$ if and only if $S > S^{ZD}$.

Since $C_{AA}(A) > 0$ and $dA/dS = d[S^{LF} - S - \delta(S, g(S))]/dS < 0$, we have that $C_A(a(S, g(S)))$ is decreasing in S for all S. Together with $C_A(a(S^{ZC}, g(S^{ZC}))) = 0$, which holds by construction, this implies that $C_A(a(s, g(S))) > 0$ if and only if $S < S^{ZC}$.

If $S^{ZD} < S^{ZC}$, C_A and D_S are both positive only for $S \in (S^{ZD}, S^{ZC})$ and have opposite signs otherwise. Hence, since $\delta_S > 0$, the equalities $\tau = C_A = D_S/(1+\delta_S)$ can only hold for positive τ. Similarly, if $S^{ZD} > S^{ZC}$, C_A and D_S are both negative only for $S \in (S^{ZD}, S^{ZC})$ and have opposite signs otherwise. Hence, the optimum arises at $\tau < 0$. This proves (i).

If $S^{\bar{T}} < \bar{S}$, then $S^{\bar{T}} < S^{ZD} < \bar{S}$ by Lemma 2.a and $D_{SG} < 0$. Similarly, if $S^{\bar{T}} > \bar{S}$, then $S^{\bar{T}} > S^{ZD} > \bar{S}$. Hence if $\max\{S^{\bar{T}}, \bar{S}\} < S^{ZC}$ then $S^{ZD} < S^{ZC}$ and it follows from (i) that $\tau > 0$. This proves (ii).

The abatement cost function implies $C_A > 0$ if and only if $A > 0$. Then (1.7) implies (iii).

Appendix B

Appendix Chapter 3

B.1 Derivations General Model

Derivation Equation (3.7): Total differentiation of equation (2) in the text with respect to g_1, g_2, a_1, and a_2 for countries 1 and 2 yields the following system of equations:

$$\begin{bmatrix} (M^1_{gg} + D^1_{GG}) & D^1_{GG} \\ D^2_{GG} & (M^2_{gg} + D^2_{GG}) \end{bmatrix} \cdot \begin{bmatrix} dg_1 \\ dg_2 \end{bmatrix} = \begin{bmatrix} -D^1_{GA} & -D^1_{GA} \\ -D^2_{GA} & -D^2_{GA} \end{bmatrix} \begin{bmatrix} da_1 \\ da_2 \end{bmatrix}$$

Using Cramer's rule obtain $dg_1/da_1 = (D^1_{GG}D^2_{GA} - D^1_{GA}(M^2_{gg} + D^2_{GG}))((M^1_{gg} + D^1_{GG})(M^2_{gg} + D^2_{GG}) - D^1_{GG}D^2_{GG})$. Divide the numerator and the denominator by $(M^1_{gg} + D^1_{GG})(M^2_{gg} + D^2_{GG})$ to obtain equation (5) in the text. Follow the same procedure to show that $dg_1/da_1 = dg_1/da_2$.

Derivation Equation (3.10): Total differentiation of (7) with respect to a_1 and a_2 yields $(C^1_{aa} + D^1_{AA} + D^1_{AG}(y^1_a + y^2_a) + (D^1_{GA} + D^1_{GG}(y^1_a + y^2_a))y^2_a + D_G y^2_{aa})da_1 = -(D^1_{AA} + D^1_{AG}(y^1_a + y^2_a) + (D^1_{GA} + D^1_{GG}(y^1_a + y^2_a))y^2_a + D_G y^2_{aa})da_2$. The left hand side of the previous equation can be rewritten as: $(C^1_{aa} + D^1_{AA} + D^1_{AG}(y^1_a + y^2_a) + D^1_{GA}(1 + \frac{D^1_{GG}/(M^1_{gg} + D^1_{GG})}{D^1_{GA}/(M^1_{gg} + D^1_{GG})}(y^1_a + y^2_a))y^2_a + D_G y^2_{aa})da_1$. Now, divide both sides of the equation by $C^1_{aa} + D^1_{AA}$ to get (12).

B.2 Proofs of Propositions

Proof Proposition 2: $a^{**} = \lim_{\gamma\to\infty} a^* = \delta R_0/(\alpha+\delta)$. The difference between the two levels of abatement is given by: $a^* - a^{**} = \delta\alpha(\gamma y_a - 2\delta)/((\alpha+\delta)(\alpha(\gamma+2\delta+2\rho) + 2(\delta\gamma+2\delta\rho+\delta\gamma y_a)))$ which is always less than zero, because y_a is less than zero. Thus, $a^* < a^{**}$ ■

Proof Proposition 3: Take equation (3.26) and replace it in equation (3.19). Evaluate at symmetry to obtain the equilibrium value of geoengineering $g^* = R_0\alpha\delta(\gamma+2(\delta+\rho))/(\alpha(\gamma+2(\delta+\rho))^2+2\delta(\gamma^2+4\rho(\delta+\rho)+\gamma(\delta+4\rho)))$. Replace the values of a^* and g^* in the equation for temperature to obtain $T^* = R_0\alpha(\gamma+2\rho)(\gamma+2(\delta+\rho))/(\alpha(\gamma+2(\delta+\rho))^2+2\delta(\gamma^2+4\rho(\delta+\rho)+\gamma(\delta+4\rho)))$. Evaluate T^* at the limit when $\gamma\to\infty$ to obtain $T^{**} = R_0\alpha/(\alpha+2\delta)$. The difference between the two temperature levels is given by: $T^* - T^{**} = 2R_0\alpha\delta(\gamma\delta-\alpha(\gamma+2(\delta+\rho))/((\alpha+2\delta)(\alpha(\gamma+2(\delta+\rho))^2+2\delta(\gamma^2+4\rho(\delta+\rho)+\gamma(\delta+4\rho)))))$ which is negative only when $\gamma\delta-\alpha(\gamma+2(\delta+\rho))<0$. Rearrange to find $\delta/(\gamma+2(\delta+\rho)) < \alpha/\gamma$ ■

Proof of Proposition 4: Replace a^*, g^*, and T^* in (3.17) to obtain $\Omega^* = R_0^2\alpha\delta(\gamma^2+4\rho(\delta+\rho)+\gamma(\delta+4\rho))(\alpha(\gamma+2(\delta+\rho))^2+\delta(\gamma^2+4\rho(\delta+\rho)+\gamma(\delta+4\rho)))/(\alpha(\gamma+2(\delta+\rho))^2+2\delta(\gamma^2+4\rho(\delta+\rho)+\gamma(\delta+4\rho)))^2$. $\Omega^{**} = \lim_{\gamma\to\infty}\Omega^* = R_0^2\alpha\delta(\alpha+\delta)/(\alpha+2\delta)^2$. The difference between the two total costs values is given by: $\Omega^*-\Omega^{**} = -R_0^2\alpha^3\delta^2(3\gamma+4(\delta+\rho))(\alpha(\gamma+2(\delta+\rho))^2+\delta(2\gamma^2+4(\delta+\rho)(\delta+2\rho)+\gamma(5\delta+8\rho)))/((\alpha+2\delta)^2(\alpha(\gamma+2(\delta+\rho))^2+2\delta(\gamma^2+4\rho(\delta+\rho)+\gamma(\delta+4\rho)))^2)$ which is always negative ■

Proof of Proposition 5: $a_1^* = R_0(\delta_2^2\rho_1+\delta_1(\gamma_2+\rho_2)^2)/(\delta_2^2\rho_1+\delta_1(\gamma_2+\rho_2)^2+\alpha_1(\gamma_2+\delta_2+\rho_2)^2)$ and $a_1^{**} = R_0\delta_1/(\alpha_1+\delta_1)$. The difference is given by $a_1^*-a_1^{**} = R_0\alpha_1\delta_2(\delta_2\rho_1-\delta_1(\delta_2+2\rho_2)-2\gamma_2\delta_1)/((\alpha_1+\delta_1)(\delta_2^2\rho_1+\delta_1(\gamma_2+\rho_2)^2+\alpha_1(\gamma_2+\delta_2+\rho_2)^2))$. The previous expression is positive when $\delta_2\rho_1-\delta_1(\delta_2+2\rho_2)-2\gamma_2\delta_1>0$ which is equivalent to

$\delta_2/\delta_1 > ((\delta_2 + 2\rho_2) + 2\gamma_2)/\rho_1$ ■

Proof of Proposition 6: The first best level of abatement is given by $a_1^{FB} = R_0(\delta_1 + \delta_2)/(\alpha_1 + \delta_1 + \delta_2)$. The difference between the level of abatement in the asymmetric equilibrium and the first best level of abatement is given by $a_1^* - a_1^{FB} = -R_0\alpha_1\delta_2(\gamma_2^2 + \delta_2(\delta_1 + \delta_2 - \rho_1) + 2(\delta_1 + \delta_2)\rho_2 + \rho_2^2 + 2\gamma_2(\delta_1 + \delta_2 + \rho_2))/((\alpha_1 + \delta_1 + \delta_2)(\delta_2^2\rho_1 + \delta_1(\gamma_2 + \rho_2)^2 + \alpha_1(\gamma_2 + \delta_2 + \rho_2)^2))$. Rearrange to obtain $\frac{\delta_1}{\delta_2} < \frac{\rho_1}{2(\gamma_2+\rho_2)+\delta_2} - \frac{(\gamma_2+\rho_2)^2/\delta_2}{2(\gamma_2+\rho_2)+\delta_2} - 1$. ■